SOLVED SUBJECTIVE EXAMPLES

Example: 1 Which of the following is the empty set 0]

(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$ (b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$

(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$ (d) $\{x : x$ is a real number and $x^2 = x + 2\}$

Example: 2 The set $A = \{x : x \in R, x^2 = 16$ and $2x = 6\}$ equals

(a) ϕ (b) [14, 3, 4] (c) [3] (d) [4]

Solution: (a) $x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations. Thus, $A = \phi$.

Example: 3 If a set A has n elements, then the total number of subsets of A is

(a) n (b) n^2 (c) 2^n (d) $2n$

Solution: (c) Number of subsets of $A = {}^nC_0 + {}^nC_1 + ... + {}^nC_n = 2^n$.

Example: 4 Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are

(a) 7, 6 (b) 6, 3 (c) 5, 1 (d) 8, 7

Solution: (b) Since $2^m - 2^n = 56 = 8 \times 7 = 2^3 \times 7 \Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times 7$

$\therefore n = 3$ and $2^{m-n} = 8 = 2^3$

$\Rightarrow m - n = 3 \Rightarrow m - 3 = 3 \Rightarrow m = 6$

$\therefore m = 6, n = 3$.

Example: 5 The number of proper subsets of the set {1, 2, 3} is

(a) 8 (b) 7 (c) 6 (d) 5

Solution: (c) Number of proper subsets of the set {1, 2, 3) = $2^3 - 2 = 6$.

Example: 6 If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n-1) : n \in N\}$, then

(a) $X \subseteq Y$ (b) $Y \subseteq X$ (c) $X = Y$ (d) None of these

Solution: (a) Since $8^n - 7n - 1 = (7+1)^n - 7n - 1 = 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + ... + {}^nC_{n-1} 7 + {}^nC_n - 7n - 1$

$= {}^nC_2 7^2 + {}^nC_3 7^3 + ... + {}^nC_n 7^n$ (${}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc.

$= 49[{}^nC_2 + {}^nC_3(7) + ... + {}^nC_n 7^{n-2}]$

$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for $n \geq 2$.

For $n = 1$, $8^n - 7n - 1 = 8 - 7 - 1 = 0$; For $n = 2$, $8^n - 7n - 1 = 64 - 14 - 1 = 49$

$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for all $n \in N$.

$\therefore$ X contains elements which are multiples of 49 and clearly Y contains all multiplies of 49.

$\therefore$ $X \subseteq Y$.

Example: 7 Given the sets $A = \{1, 2, 3\}, B = \{3, 4\}$, C = {4, 5, 6}, then $A \cup (B \cap C)$ is

(a) {3} (b) {1, 2, 3, 4} (c) {1, 2, 4, 5} (d) {1, 2, 3, 4, 5, 6}

Solution: (b) $B \cap C = \{4\}$, $\therefore$ $A \cup (B \cap C)$ = {1, 2, 3, 4}.

Example: 8 If $A \subseteq B$, then $A \cup B$ is equal to

(a) A (b) $B \cap A$ (c) B (d) None of these

Solution: (c) Since $A \subseteq B \Rightarrow A \cup B = B$.

Example: 9 If A and B are any two sets, then $A \cup (A \cap B)$ is equal to **[Karnataka CET 1996]**

(a) A (b) B (c) A^c (d) B^c

Solution: (a) $A \cap B \subseteq A$. Hence $A \cup (A \cap B) = A$.

Example: 10 If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

(a) A (b) B (c) ϕ (d) $A \cap B^c$

Solution: (d) $A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$ = $(A \cap A^c) \cup (A \cap B^c)$ = $\phi \cup (A \cap B^c) = A \cap B^c$.

Example: 11 If $N_a = \{an : n \in N\}$, then $N_3 \cap N_4 =$

(a) N_7 (b) N_{12} (c) N_3 (d) N_4

Solution: (b) $N_3 \cap N_4 = \{3, 6, 9, 12, 15 ......\} \cap \{4, 8, 12, 16, 20, ...\}$

= {12, 24, 36......} = N_{12}

Trick: $N_3 \cap N_4 = N_{12}$ [3, 4 are relatively prime numbers]

Example: 12 If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where b, $c \in N$ are relatively prime, then **[DCE 1999]**

(a) $d = bc$ (b) $c = bd$ (c) $b = cd$ (d) None of these

Solution: (a) $bN =$ the set of positive integral multiples of b, cN = the set of positive integral multiplies of c.

$\therefore$ $bN \cap cN$ = the set of positive integral multiples of bc = $b \subset N$ [$\because b,c$ are prime]

$\therefore$ $d = bc$.

Example: 13 If the sets A and B are defined as

$A = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$

$B = \{(x, y) : y = -x, x \in R\}$, then

(a) $A \cap B = A$ (b) $A \cap B = B$ (c) $A \cap B = \phi$ (d) None of these

Solution: (c) Since $y = \frac{1}{x}, y = -x$ meet when $-x = \frac{1}{x}$ $\Rightarrow$ $x^2 = -1$, which does not give any real value of x

Hence $A \cap B = \phi$.

Example: 14 Let $A = [x : x \in R, |x| < 1]$; $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is

(a) $[x : 1 < x \leq 2]$ (b) $[x : 1 \leq x < 2]$ (c) $[x : 1 \leq x \leq 2]$ (d) None of these

Solution: (b) $A = [x : x \in R, -1 < x < 1]$

$B = [x : x \in R : x - 1 \leq -1$ or $x - 1 \geq 1]$ = $[x : x \in R : x \leq 0 \text{ or } x \geq 2]$

$\therefore$ $A \cup B = R - D$

Where D = $[x : x \in R, 1 \leq x < 2]$

Example: 15 If the sets A and B are defined as

$A = \{(x, y) : y = e^x, x \in R\}$

$B = \{(x, y) : y = x, x \in R\}$, then

(a) $B \subseteq A$ (b) $A \subseteq B$ (c) $A \cap B = \phi$ (d) $A \cup B = A$

Solution: (c) Since, $y = e^x$ and $y = x$ do not meet for any $x \in R$

$\therefore$ $A \cap B = \phi$.

Example: 16 If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, then $X \cup Y$ is equal to

(a) X (b) Y (c)N (d)None of these

Solution: (b) Since, $4^n - 3n - 1 = (3+1)^n - 3n - 1$ = $3^n + {}^nC_1 3^{n-1} + {}^nC_2 3^{n-2} + \ldots + {}^nC_{n-1} 3 + {}^nC_n - 3n - 1$

= ${}^nC_2 3^2 + {}^nC_3 . 3^3 + \ldots + {}^nC_n 3^n$ $({}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc.

= $9[{}^nC_2 + {}^nC_3 (3) + \ldots + {}^nC_n 3^{n-1}]$

$\therefore$ $4^n - 3n - 1$ is a multiple of 9 for $n \geq 2$.

For $n = 1$, $4^n - 3n - 1 = 4 - 3 - 1 = 0$, For $n = 2$, $4^n - 3n - 1 = 16 - 6 - 1 = 9$

$\therefore$ $4^n - 3n - 1$ is a multiple of 9 for all $n \in N$

$\therefore$ X contains elements which are multiples of 9 and clearly Y contains all multiples of 9.

$\therefore$ $X \subseteq Y$, $\therefore$ $X \cup Y = Y$.

***Example* 17:**

In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution:

Let A = set of people who like coffee

B = set of people who like tea

Given, $n(A \cup B) = 70$, $n(A) = 37$, $n(B) = 52$

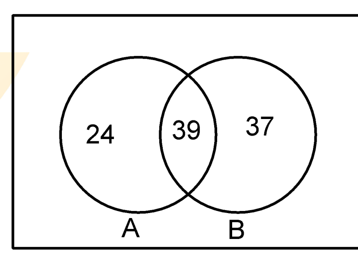
To find $n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$ = 37 + 52 – 70 = 19

***Example* 18:**

If 53% of persons like oranges where 66% like apples, what can be said about the percentage of persons who like both oranges and apples?

Solution: Let the total number of persons = 100 ⇒ $n(A \cup B) = 100$



Let A = {x : x like oranges}

B = {x : x likes apples)

Then $n(A) = 53$, $n(B) = 66$

$\therefore$ $A \cap B = \{x : x \text{ likes oranges and apples both}\}$

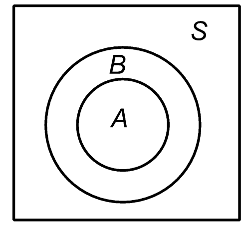
Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore$ $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 53 + 66 - 100 = 19$

***Example* 19:**

Let *A* has 3 elements and *B* has 6 elements. What can minimum number of elements in $A \cup B$?

Solution: Clearly $A \cup B$ will contain minimum number of elements if $A \subseteq B$ or $B \subseteq A$



But $n(A) = 3 < 6 = n(B)$

$\therefore \quad B \not\subseteq A \quad \therefore \; A \subset B$

Thus $A \cup B = B \quad \therefore \quad n(A \cup B) = n(B) = 6$

Thus $A \cup B$ contains at least 6 elements

Example **20**:

In a group of 2000 people, there are 1500, who can speak Hindi and 800, who can speak Bengali. How many can speak Hindi only? How many can speak Bengali only? How many can speak both Hindi and Bengali?

Solution: Let $A = \{x : x \text{ speaks Hindi}\}, \; B = \{x : x \text{ speaks Bengali}\}$

Then $A - B = \{ x : x$ speaks Hindi and can not speak Bengali}

$B - A$ = {$x : x$ speaks Bengali and can not speak Hindi}

$A \cap B$ = {$x : x$ speaks Hindi and Bengali both)

Given, $n(A) = 1500, \; n(B) = 800, \; n(A \cup B) = 2000$

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 1500 + 800 - 2000 = 300$

$\therefore$ Number of people speaking Hindi and Bengali both is 300

$n(A) = n(A - B) + n(A \cap B) \Rightarrow$

$n(A - B) = n(A) - n(A \cap B) = 1500 - 300 = 1200$

Also, $n(B - A) = n(B) - n(A \cap B) = 800 - 300 = 500$

Thus number of people speaking Hindi only = 1200

And number of people speaking Bengali only = 500

Example **21`**:

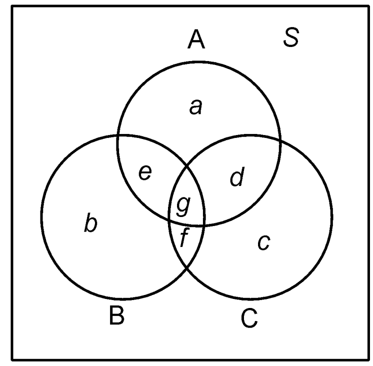
A class has 175 students. Following is the description showing the number of students studying one or more of the following subjects in this class.

Mathematics 100, Physics 70, Chemistry 46; Physics and Chemistry 23; Mathematics and Physics 30; Mathematics and Chemistry 28; Mathematics, Physics and Chemistry 18.

How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not offered any of these three subjects.

Solution: Let *A, B* and *C* denote the sets of students studying Mathematics, Physics and Chemistry respectively.

Let us denote the number of elements (students) contained in the bounded region as shown in the diagram by *a, b, c, d, e, f* and *g* respectively.



Then $a+d+g+e=100$

$b+f+g+e=70$

$c+f+g+d=46$

$g+e=30$

$g+d=28$

$g+f=23$

$g = 18$

Solving these, we get

$g=18,\ f=5,\ d=10,\ e=12,\ c=13,\ b=35,\ a=60$

$\therefore \quad a+b+c+d+e+f+g=153$

So, the number of students who have not offered any these three subjects

175 – 153 = 22

Students studying Mathematics only = *a* = 60

Students studying Physics only = *b* = 35

Students studying Chemistry only = *c* = 13

***Example* 22:**

For any sets *A, B, C.* Using logical method, prove that

(i) $A \cap (B - A) = \phi$ **(ii)** $A - B = B - A \Leftrightarrow A = B$

(iii) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ **(iv)** $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Solution: (i) Let $x \in A \cap (B - A) \Rightarrow x \in A$ and $x \in (B - A)$

$\Rightarrow x \in A$ and $(x \in B \text{ and } x \notin A)$

$\Rightarrow (x \in A \text{ and } x \in B)$ and $(x \in A \text{ and } x \notin A)$

$\Rightarrow \quad x \in (A \cap B)$ and $x \in \phi$

$\Rightarrow \quad x \in \phi$ [∵ φ has no element]

Hence $A \cap (B - A) \subseteq \phi$ ...(i)

But φ is a subset of each set.

$\therefore \quad \phi \subseteq A \cap (B - A)$ ...(ii)

From (i) and (ii), we have, $A \cap (B - A) = \phi$

(ii) $A - B = B - A \Leftrightarrow A = B$

Only if part: Let $A - B = B - A$ ...(i)

To prove $A = B$

Let $x \in A \Leftrightarrow (x \in A \text{ and } x \notin B)$ or $(x \in A \text{ and } x \in B)$

$\Leftrightarrow x \in (A - B)$ or $x \in (A \cap B)$

$\Leftrightarrow x \in (B - A)$ or $x \in A \cap B$ [from (i)]

$\Leftrightarrow (x \in B \text{ and } x \notin A)$ or $(x \in B \text{ and } x \in A)$

$\Leftrightarrow x \in B$

Hence $A = B$

If part: Let $A = B$ ...(ii)

To prove $A - B = B - A$

Now, $A - B = A - A = \phi$ [∵ $B = A$]

and $B - A = A - A = \phi$ [∵ $B = A$]

$\therefore \quad A - B = B - A$

Thus $A = B \Rightarrow A - B = B - A$

(iii) $x \in (A \cup B) - (A \cap B)$

$\Leftrightarrow \quad x \in (A \cup B) \wedge x \notin (A \cap B)$ [$\wedge$ stands for 'and']

$\Leftrightarrow \quad (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)$ [$\vee$ stands for 'or']

$\Leftrightarrow \quad [(x \in A \vee x \in B) \wedge (x \notin A)] \vee [(x \in A \vee x \in B) \wedge x \notin B]$

$\Leftrightarrow \quad [x \in B - A] \vee [x \in A - B]$

$\Leftrightarrow \quad x \in (B - A) \cup (A - B)$

$\Leftrightarrow \quad x \in (A - B) \cup (B - A)$ [∵ $A \cup B = B \cup A$]

Thus $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

(iv) $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) or $(x \notin A$ and $x \in B)$ or $(x \in A$ or $x \in B)$

$\Leftrightarrow x \in A - B$ or $x \in B - A$ or $x \in A \cap B$

$\Leftrightarrow x \in (A - B) \cup (B - A) \cup (A \cap B)$

$\therefore \quad A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$