SOLVED SUBJECTIVE EXAMPLES

Example: 1 Let $X=\{1,2,3,4,5\}$ and $Y=\{1,3,5,7,9\}$. Which of the following is/are relations from

X to *Y*

(a) $R_1=\{(x,y)\mid y=2+x, x\in X, y\in Y\}$ (b) $R_2=\{(1,1),(2,1),(3,3),(4,3),(5,5)\}$

(c) $R_3=\{(1,1),(1,3)(3,5),(3,7),(5,7)\}$ (d) $R_4=\{(1,3),(2,5),(2,4),(7,9)\}$

Solution: (a,b,c) R_4 is not a relation from *X* to *Y*, because (7, 9) $\in R_4$ but (7, 9) $\notin X\times Y$.

Example: 2 Given two finite sets *A* and *B* such that *n*(*A*) = 2, *n*(*B*) = 3. Then total number of relations from *A* to *B* is

(a) 4 (b) 8 (c) 64 (d) None of these

Solution: (c) Here $n(A\times B)$ = 2 × 3 = 6

Since every subset of *A* × *B* defines a relation from *A* to *B,* number of relation from *A* to *B* is equal to number of subsets of $A\times B=2^6=64,$ which is given in (c).

Example: 3 The relation *R* defined on the set of natural numbers as {(*a*, *b*) : *a* differs from *b* by 3}, is

given by

(a) {(1, 4, (2, 5), (3, 6),..... (b){ (4, 1), (5, 2), (6, 3),.....}

(c) {(1, 3), (2, 6), (3, 9),..} (d) None of these

Solution: (b) $R=\{(a,b): a,b\in N, a-b=3\}$ = $\{((n+3),n): n\in N\}=\{(4,1),(5,2),(6,3).....\}$

Example: 4 Let *A* = {1, 2, 3}, *B* = {1, 3, 5}. *A* relation $R: A\to B$ is defined by *R* = {(1, 3), (1,

5), (2, 1)}. Then R^{-1} is defined by

(a) {(1,2), (3,1), (1,3), (1,5)} (b) {(1, 2), (3, 1), (2, 1)} (c) {(1, 2), (5, 1), (3, 1)} (d) None of these

Solution: (c) $(x,y)\in R \Leftrightarrow (y,x)\in R^{-1}$, $\therefore$ $R^{-1}=\{(3,1),(5,1),(1,2)\}$.

Example: 5 The relation *R* is defined on the set of natural numbers as {(*a*, *b*) : *a* = 2*b*}.

Then R^{-1} is

given by

(a) {(2, 1), (4, 2), (6, 3).....} (b) {(1, 2), (2, 4), (3, 6)....} (c) R^{-1} is not defined (d) None of these

Solution: (b) R = {(2, 1), (4, 2), (6, 3),......} So, R^{-1} = {(1, 2), (2, 4), (3, 6),.....}.

Example: 6 The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is

(a) Reflexive but not symmetric (b) Reflexive but not transitive

(c) Symmetric and Transitive (d) Neither symmetric nor transitive

Solution: (a) Since (1, 1); (2, 2); (3, 3) ∈ R therefore R is reflexive. (1, 2) ∈ R but (2, 1) ∉ R, therefore R is not symmetric. It can be easily seen that R is transitive.

Example: 7 Let R be the relation on the set R of all real numbers defined by $a\ R\ b$ iff $|a-b|\le 1$. Then R is

(a) Reflexive and Symmetric (b) Symmetric only (c) Transitive only (d) Anti-symmetric only

Solution: (a) $|a-a|=0<1 \quad \therefore aRa \forall a \in R$

$\therefore$ R is reflexive, Again $a\ R\ b$ ⇒ $|a-b|\le 1 \Rightarrow |b-a|\le 1 \Rightarrow bRa$

$\therefore$ R is symmetric, Again $1R\frac{1}{2}$ and $\frac{1}{2}R1$ but $\frac{1}{2}\ne 1$

$\therefore$ R is not anti-symmetric

Further, 1 R 2 and 2 R 3 but 1 R 3

[$\because |1-3|=2>1$]

$\therefore$ R is not transitive.

Example: 8. The relation "less than" in the set of natural numbers is

(a) Only symmetric (b) Only transitive

(c) Only reflexive (d) Equivalence relation

Solution: (b) Since $x<y, y<z \Rightarrow x<z \forall x,y,z \in N$

$\therefore$ $xRy, yRz \Rightarrow xRz$, $\therefore$ Relation is transitive, $\therefore$ $x<y$ does not give $y<x$, $\therefore$ Relation is not symmetric.

Since $x<x$ does not hold, hence relation is not reflexive.

Example: 9 With reference to a universal set, the inclusion of a subset in another, is relation, which is

(a) Symmetric only (b) Equivalence relation

(c)Reflexive only (d) None of these

Solution: (d) Since $A \subseteq A$ $\therefore$ relation '$\subseteq$' is reflexive.

Since $A \subseteq B,\ \ B \subseteq C \Rightarrow A \subseteq C$

$\therefore$ relation '$\subseteq$' is transitive.

But $A \subseteq B,$ $\Rightarrow$ $B \subseteq A$, $\therefore$ Relation is not symmetric.

Example: 10 Let $A = \{2,4,6,8\}$. A relation *R* on *A* is defined by $R = \{(2,4),(4,2),(4,6),(6,4)\}$. Then *R*

is

(a) Anti-symmetric (b) Reflexive (c) Symmetric (d) Transitive

Solution: (c) Given *A* = {2, 4, 6, 8}

R = {(2, 4)(4, 2) (4, 6) (6, 4)}

(*a*, *b*) ∈ *R* ⇒ (*b*, *a*) ∈ *R* and also $R^{-1} = R$. Hence *R* is symmetric.

Example: 11 Let $P = \{(x,y)|\ x^2 + y^2 = 1, x, y \in R\}$. Then *P* is

(a) Reflexive (b)Symmetric (c)Transitive (d)Anti-symmetric

Solution: (b) Obviously, the relation is not reflexive and transitive but it is symmetric,

because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

Example: 12. Let *R* be a relation on the set *N* of natural numbers defined by *nRm* ⇔ *n* is a factor of *m* (*i.e., n*|*m*). Then *R* is

(a) Reflexive and symmetric (b)Transitive and symmetric

(c) Equivalence (d)Reflexive, transitive but not symmetric

Solution: (d) Since *n* | *n* for all $n \in N$, therefore *R* is reflexive. Since 2 | 6 but $6 | 2$, therefore

R is not symmetric.

Let *n R m* and *m R p* ⇒ *n*|*m* and *m*|*p* ⇒ *n*|*p* ⇒ *nRp*. So *R* is transitive.

Example: 13 Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

(a) Less than n (b)Greater than or equal to n

(c) Less than or equal to n (d) None of these

Solution: (b) Since R is an equivalence relation on set A, therefore $(a, a) \in R$ for all $a \in A$. Hence, R has at least n ordered pairs.

Example: 14 Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$ if $ad(b+c) = bc(a+d)$, then R is

[Roorkee 1995]

(a) Symmetric only (b) Reflexive only (c) Transitive only (d) An equivalence relation

Solution: (d) For $(a, b), (c, d) \in N \times N$

$(a,b)R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

Reflexive: Since $ab(b+a) = ba(a+b) \forall ab \in N$,

$\therefore$ $(a,b)R(a,b)$, $\therefore$ R is reflexive.

Symmetric: For $(a,b),(c,d) \in N \times N$, let $(a,b)R(c,d)$

$\therefore$ $ad(b+c) = bc(a+d)$ $\Rightarrow$ $bc(a+d) = ad(b+c)$ $\Rightarrow$ $cb(d+a) = da(c+b)$ $\Rightarrow$ $(c,d)R(a,b)$

$\therefore$ R is symmetric $(a,b),(c,d),(e,f) \in N \times N,$

Transitive: For Let $(a,b)R(c,d),(c,d)R(e,f)$

$\therefore$ $ad(b+c) = bc(a+d)$, $cf(d+e) = de(c+f)$

$\Rightarrow$ $adb + adc = bca + bcd$ .....(i) and $cfd + cfe = dec + def$ ....(ii)

(i) × ef + (ii) × ab gives, $adbef + adcef + cfdab + cfeab$ = $bcaef + bcdef + decab + defab$

$\Rightarrow$ $adcf(b+e) = bcde(a+f)$ $\Rightarrow$ $af(b+e) = be(a+f)$ $\Rightarrow$ $(a,b)R(e,f)$. $\therefore$ R is transitive. Hence R is an equivalence relation.

Example: 15 For real numbers x and y, we write $x\,Ry \Leftrightarrow x - y + \sqrt{2}$ is an irrational number.

Then the relation R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

Solution: (a) For any $x \in R,$ we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

$\Rightarrow$ xRx for all x. So, R is reflexive.

R is not symmetric, because $\sqrt{2}R1$ but $1 R \sqrt{2}$, R is not transitive also because $\sqrt{2}$ R 1 and $1R2\sqrt{2}$ but $\sqrt{2} R 2\sqrt{2}$.

Example: 16 Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is

(a) Reflexive (b) Symmetric (c) Anti-symmetric (d) Transitive

Solution: (b) Clearly, the relation is symmetric but it is neither reflexive nor transitive.

Example: 17 Let R and S be two non-void relations on a set A. Which of the following statements is false

(a) R and S are transitive ⇒ $R \cup S$ is transitive (b) R and S are transitive ⇒ $R \cap S$ is transitive

(c) R and S are symmetric ⇒ $R \cup S$ is symmetric (d) R and S are reflexive ⇒ $R \cap S$ is reflexive

Solution: (a) Let $A = \{1, 2, 3\}$ and R = {(1, 1), (1, 2)}, S = {(2, 2) (2, 3)} be transitive relations on A.

Then $R \cup S$ = {(1, 1); (1, 2); (2, 2); (2, 3)}

Obviously, $R \cup S$ is not transitive. Since (1, 2) $\in$ $R \cup S$ and $(2, 3) \in R \cup S$ but (1, 3) $\notin R \cup S$.

Example: 18 The solution set of $8x \equiv 6(\text{mod } 14), x \in Z$, are

(a) [8] ∪ [6] (b) [8] ∪ [14] (c) [6] ∪ [13] (d) [8] ∪ [6] ∪ [13]

Solution: (c) $8x - 6 = 14P (P \in Z)$ $\Rightarrow$ $x = \frac{1}{8}[14P + 6], x \in Z$

$\Rightarrow x = \frac{1}{4}(7P + 3)$ $\Rightarrow$ x = 6, 13, 20, 27, 34, 41, 48,.......

$\therefore$ Solution set = {6, 20, 34, 48,.....} ∪ {13, 27, 41, ...} = [6] ∪ [13].

Where [6], [13] are equivalence classes of 6 and 13 respectively.

Example: 19 If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR

(a) Is from A to C (b) Is from C to A

(c)Does not exist (d) None of these

Solution: (a) It is obvious.

Example: 20 If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(SoR)^{-1} =$

(a) $S^{-1}oR^{-1}$ (b) $R^{-1}oS^{-1}$ (c) SoR (d) RoS

Solution: (b) It is obvious.

Example: 21 If R be a relation < from A = {1,2, 3, 4} to B = {1, 3, 5} *i.e.,* $(a,b) \in R \Leftrightarrow a < b$, then

RoR^{-1} is

(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}

(b) {(3, 1) (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}

(c) {(3, 3), (3, 5), (5, 3), (5, 5)}

(d) {(3, 3) (3, 4), (4, 5)}

Solution: (c) We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)}

$R^{-1} =$ {(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)}

Hence RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

Example: 22 Let a relation R be defined by R = {(4, 5); (1, 4); (4, 6); (7, 6); (3, 7)} then

$R^{-1}oR$ is

(a) {(1, 1), (4, 4), (4, 7), (7, 4), (7, 7), (3, 3)}

(b) {(1, 1), (4, 4), (7, 7), (3, 3)}

(c) {(1, 5), (1, 6), (3, 6)}

(d)None of these

Solution: (a) We first find R^{-1}, we have $R^{-1} = \{(5,4);(4,1);(6,4);(6,7);(7,3)\}$ we now obtain the elements of $R^{-1}oR$ we first pick the element of R and then of R^{-1}. Since $(4,5) \in R$ and $(5,4) \in R^{-1}$, we have $(4,4) \in R^{-1}oR$

Similarly, $(1,4)\in R,(4,1)\in R^{-1}\Rightarrow(1,1)\in R^{-1}oR$

***Example* 23:**

Solve $0<|x-1|\le 3$ **for real values of *x*.**

Solution:

Here $|x-1|>0$

$\Rightarrow$ $x\ne 1$ ...(i)

and $|x-1|\le 3$ $\Rightarrow$ $-3\le x-1\le 3$

$\Rightarrow$ $-2\le x\le 4$ ...(ii)

Combing (i) and (ii) $\Rightarrow$ $x\in[-2,\ 1)\cup(1,\ 4]$

***Example* 24:**

Solve $|x-1|+|2x-3|=|3x-4|$**.**

Solution:

Since $3x-4=(x-1)+(2x-3)$

$\Rightarrow$ $|3x-4|=|x-1|+|2x-3|$

$\Rightarrow$ $(x-1)(2x-3)\ge 0$

$\Rightarrow$ $x\in(-\infty,\ 1]\cup\left[\frac{3}{2},\ \infty\right)$

***Example* 25:**

Let $f(x)=\sqrt{x+3}$ **and** $g(x)=\sqrt{3-x^2}$ **, then find the domain for**

(i) $f+g$ **(ii)** $\frac{f}{g}$

Solution:

For domain of f : $x+3\ge 0$ $\Rightarrow$ $x\ge -3$ $\Rightarrow$ $[-3,\ \infty)$ ...(i)

For domain of g : $3-x^2\ge 0$

$\Rightarrow \quad (3-x)(3+x) \geq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$ ...(ii)

(i) $f+g = \sqrt{x+3} + \sqrt{3-x^2}$

Domain of $(f+g) =$ domain of $(f) \cap$ domain of (g)

$= [-3, \infty) \cap [-3, 3]$

$= [-3, 3]$

(ii) For $\frac{f}{g}$

Here $g(x) \neq 0 \Rightarrow 3 - x^2 \neq 0 \Rightarrow x \neq \pm 3$

$\therefore \quad \frac{f}{g} = \frac{\sqrt{x+3}}{\sqrt{3-x^2}} = \frac{1}{\sqrt{3-x}}$

Domain of $\frac{f}{g}$ =domain of $(f) \cap$ domain of (g) $-\{-3, 3\}$

$= (-3, 3)$

***Example* 26:**

Find the domain of $f(x) = \frac{1}{\sqrt{x+|x|}}$.

Solution:

$|x|$ is defined as

$$|x| = \begin{cases} x & , \quad x \geq 0 \\ -x & , \quad x < 0 \end{cases}$$

$$\Rightarrow \quad x + |x| = \begin{cases} x + x = 2x & , \quad x \geq 0 \\ x - x = 0 & , \quad x < 0 \end{cases}$$

$f(x)$ is defined for $x + |x| > 0$

$\Rightarrow \quad x > 0$

$\Rightarrow \quad x \in (0, \infty)$

$\therefore$ Domain of $f(x)$ is $(0, \infty)$

***Example* 27:**

Find the domain of the function $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$ **.**

Solution:

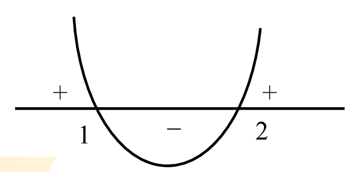
$f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$

$f(x)$ is defined as $\dfrac{1-|x|}{2-|x|} \geq 0$ provided $|x| \neq 2 \Rightarrow x \neq \pm 2$ ...(i)

$\Rightarrow \quad \dfrac{|x|-1}{|x|-2} \geq 0$

Let $|x| = t$

$\Rightarrow \quad \dfrac{t-1}{t-2} \geq 0$



$\Rightarrow \quad t \leq 1$ or $t \geq 2$

$\Rightarrow \quad |x| \leq 1$ or $|x| \geq 2$

$\Rightarrow \quad x \in [-1, 1]$ or $x \in (-\infty, -2] \cup [2, \infty)$ ...(ii)

Combining (i) and (ii) $\Rightarrow$ domain of $f = (-\infty, -2) \cup (2, \infty) \cup [-1, 1]$

***Example* 28:**

Solve $\left|\dfrac{2}{x-3}\right| > 1$, $x \neq 3$ **.**

Solution:

$\left|\frac{2}{x-3}\right|>1 \Rightarrow \frac{2}{|x-3|}>1 \Rightarrow 2>|x-3| \Rightarrow |x-3|<2$

$\Rightarrow -2<x-3<2 \Rightarrow 1<x<5$, but $x\neq 3 \Rightarrow x\in(1,\ 3)\cup(3,\ 5)$

***Example* 29:**

Solve $\frac{-1}{|x|-2}\geq 1,\ x\neq \pm 2$.

Solution:

$\frac{-1}{|x|-2}\geq 1 \Rightarrow \frac{-1}{|x|-2}-1\geq 0 \Rightarrow \frac{-1-(|x|-2)}{|x|-2}\geq 0$

$\Rightarrow \frac{1-|x|}{|x|-2}\geq 0 \Rightarrow \frac{|x|-1}{|x|-2}\leq 0 \Rightarrow 1\leq |x|\leq 2 \Rightarrow x\in(-2,\ -1]\cup[1,\ 2]$ but $x\neq \pm 2$

$\Rightarrow x\in(-2,\ -1]\cup[1,\ 2)$

***Example* 30:**

Find the domain of the function $y = f(x)$ **given by** $10^x + 10^y = 10$.

Solution:

$10^x + 10^y = 10 \quad \Rightarrow \quad 10^y = 10 - 10^x \quad \Rightarrow \quad y = \log_{10}(10 - 10^x) \Rightarrow \quad 10 - 10^x > 0$

$\Rightarrow \quad 10 > 10^x \quad \Rightarrow x < 1$

$\Rightarrow$ Domain is $(-\infty, 1)$

***Example* 31:**

Find the domain of $f(x) = \log_5 \log_5 (1 + x^3)$.

Solution:

$f(x) = \log_5 \log_5 (1 + x^3)$

$\Rightarrow \quad \log_5(1 + x^3) > 0 \quad \Rightarrow \quad 1 + x^3 > 5^0 \quad \Rightarrow \quad 1 + x^3 > 1 \quad \Rightarrow \quad x^3 > 0$

$\Rightarrow \quad x \in (0, \infty)$ ...(i)

Also $1 + x^3 > 0$

$x^3 > -1 \Rightarrow \quad x > -1$ ...(ii)

Combining (i) and (ii) $\Rightarrow$ Domain is $(0, \infty)$

***Example* 32:**

Find the domain of the function $f(x) = \dfrac{\log_2(x+3)}{x^2 + 3x + 2}$.

Solution:

$f(x) = \dfrac{\log_2(x+3)}{x^2 + 3x + 2} = \dfrac{\log_2(x+3)}{(x+1)(x+2)}$

$\Rightarrow \quad x + 3 > 0 \quad \Rightarrow \quad x > -3$ ...(i)

And $(x+1)(x+2) \neq 0 \quad \Rightarrow \quad x \neq -1, -2$ ...(ii)

Combining (i) and (ii) $\Rightarrow$ $x \in (-3, \infty) - \{-1, -2\}$