LESSON 1

SETS

1. SET-DEFINITION

A set is a well-defined class or collection of objects. By a well defined collection we mean that there exists a rule with the help of which it is possible to tell whether a given object belongs or does not belong to the given collection. The objects in sets may be anything, numbers, people, mountains, rivers etc. The objects constituting the set are called elements or members of the set.

A set is a collection of well-defined distinct objects i.e. the objects follow a given rule or rules.

If we say that we have a collection of short students in a class, then this collection is not a set as short student is not well defined. If however, we say that we have a collection of students whose heights is less then 5 feet, then it represent a set.

Examples:

1. $A = \{1, 4, 5, 4, 8\}$, the elements of this collection are distinguishable but not distinct, hence *A* is not a set.

2. Let $A =$ collection of all vowels in English alphabets, then $A = \{a, e, i, o, u\}$. Hence elements of *A* are distinguishable as well as distinct, then *A* is a set.

3. The collection of all positive integers is a set.

4. The collection of all students of IIT (Delhi) is a set.

Some standard notation for some special sets:

1. The set of all natural number i.e., the set of all positive integers, is denoted by *N*.
2. The set of all integer number is denoted by *I* or *Z.*
3. The set of rational number is denoted by *Q*.
4. The set of all irrational number is denoted by Q'.

5. The set of all real number is denoted by R.
6. The set of all positive number is denoted by R^+. (zero is not included)
7. The set of all negative real number is denoted by R^-. (zero is not included)
8. The set of complex number is denoted by C.

1.1 REPRESENTATION OF SETS

- **Tabular form or Roster form**

In this method of describing a set, the elements of the set are listed separated by coma within braces.

Example: The set of prime number less then 10 can be described as {2, 3, 5, 7}

- **Set Builder form or Rule method**

In this method of describing a set, a variable x which stands for each element of the set is written under braces and then after giving a semicolor or oblique line the property or properties $P(x)$ possessed by each element of set is written the braces itself.

Example1: The set A of all even natural number can be written as $A = \{2x : x \in N\}$

Example2: The set $A = \{1, 3, 5\}$ can be written as $A = \{x$: x is an odd natural number ≤ 5}

1.2 FINITE AND INFINITE SETS

A set having finite number of elements is called a **finite set**.

Example: A = {1, 2, 3, 4}. A is a finite set as it contains 4 elements.

A set which is not a finite set is called an **infinite set**. Thus a set A is said to be an infinite set if the number of elements of set A is not finite.

Example: Let A = set of all points on a particular straight line.

1.3 CARDINAL NUMBER OF A FINITE SET

The number of elements in a finite set A is called the cardinal number of set A and is denoted by $n(A)$

Example: Let A = {1, 2, 3, 4, 5}, then $n(A)$ = 5

1.4 EQUIVALENT SETS

Two finite sets A and B are said to be equivalent if they have the same cardinal number. Thus set A and B are equivalent iff $n(A) = n(B)$.

If sets A and B are equivalent, we write $A \approx B$

Example: Let A = {1, 2, 3, 4, 5}, B = {a, e, i, o, u}

Here $n(A) = n(B)$ = 5

Therefore, sets A and B are equivalent.

1.5 EQUAL SETS

Two set A and B are said to be equal set if each element of set *A* is an element of set *B* and each element of B is an element of set *A*. Thus two sets A and B are equal if they have exactly the same elements. The order in which the elements in the two sets have been written is immaterial.

If set A and B are equal we can write A = B

Example1: Let $A = \{1, 2, 3, 4, 5\}$, $B = \{ x : x \in N$ and $1 \le x \le 5 \}$

Here *A* and *B* are equal.

2. DIFFERENT TYPES OF SETS

2.1 NULL SET (OR EMPTY SET OR VOID A SET)

A set having no element is called null set or empty set or void set. It is denoted by φ or { }.

Example: The set of odd numbers divisible by 2.

2.2 SINGLETON SET

A set having single element is called a singleton set. It is represented by writing down the element within the braces.

Example: {2}, {0}, {φ}.

2.3 UNIVERSAL SET

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by *U*.

2.4 PAIR SET

A set having two elements is called a pair set.

Example: {1, 2}, {2, 0}.

2.5 SET OF SETS

A set *S* having all its elements as set is called a set of sets or a family of sets or a class of sets.

Example1: $S = \{\{1, 2, 3\}, 3, \{4\}\}$ is not a set of sets as 3 is not a set.

Example2: {φ} is a singleton set of set having null set φ as its elements.

3. SUBSETS, SUPERSETS, PROPER SUBSETS

3.1 SUBSETS OF A SET

A set A is said to be a subset of a set *B* if each element of *A* is also an element of *B*. If *A* is a subset of set *B*, we write $A \subseteq B$

Thus, $A \subseteq B \iff [x \in A \Rightarrow x \in B]$

Example: Let *A* = {1, 2, 3}, *B* = {2, 3, 4, 1, 5}, then $A \subseteq B$.

The statement $A \subseteq B$ can also be expressed equivalently by writing $B \supseteq A$ (read '*B* is a superset of *A*')

If *A* is not a subset of *B* i.e., if there is an element in *A* which is not an element of *B*, then we write $A \not\subseteq B$ or $B \not\supseteq A$.

- **Some important properties of subset**
  - Every set is its own subset.

    Let *A* be any set ; $x \in A \Rightarrow x \in A$

    Hence $A \subseteq A$
  - Empty set is a subset of each set.
  - Let *A* and *B* be any two sets:

    then $A = B \iff A \subseteq B$ and $B \subseteq A$
  - Let *A, B, C* be three sets.

    If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

3.2 PROPER SUBSET OF A SET

A set *A* is said to be a proper subset of a set *B,* if *A* is a subset of *B* and $A \neq B$ i.e. if

Every element of *A* is an elements of *B* and *B* has at least one element which is not an **element of *A*. This fact is expressed by writing** $A \subset B$ **or** $B \supset A$**.**

If A is not a proper subset of *B*, then we write $A \not\subset B$.

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 1, 5\}$, then $A \subset B$ and $B \supset A$.

3.3 SUPERSET OF SETS

A set *A* is said to be a super set of set *B*, if *B* is a subset of *A* i.e., each elements of *B* is an elements of A. If A is a super set of B, then $A \supseteq B$.

Example: Let A = {1, 2, 3, 4, 5) and B = {2, 5, 4}.

Here B is a subset of A, therefore A is a superset of B.

3.4 POWER SET

The set or family of all the subsets of a given set A is said to be the power set of A and is **denoted by** $P(A)$

Example: If A = {1, 2}

$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

If A has n elements then $P(A)$ has 2^n elements.

Illustration 1

Question: **List all the subsets and all the proper subsets of the set {–1, 0, 1}.**

Solution: Let $A = \{-1, 0, 1\}$.

Subset of A having no element is : φ

Subsets of A having one element are : {–1}, {0}, {1}.

Subsets of A having two elements are : {–1, 0}, {0, 1}, {–1, 1}.

Subsets of A having three elements are : {–1, 0, 1}.

Thus, all the subsets of A are φ, {–1}, {0}, {1}, {–1, 0}, {0, 1}, {–1, 1}, {–1, 0, 1}.

Proper subsets of A are φ, {–1}, {0}, {1}, {–1, 0}, {0, 1}, {–1, 1}.

Illustration 2

Question: **Make correct statements by filling the blanks by suitable symbols $\subseteq, \not\subseteq$.**

(i) {*x* : *x* is an even natural number} ____ {*x* : *x* is an integer}
(ii) {*x* : *x* is a triangle in the plane} ____ {*x* : *x* is a rectangle in the plane}
(iii) {*x* : *x* is isosceles triangle in the plane} ____ {*x* : *x* is an equilateral triangle in the plane}
(iv) *a* ____ {*a*, {*b*}, *c*}
(v) $\{\{a\}\}$ ____ $\{a, \{b\}, c\}$

Solution: (i) Since every even natural number is an integer, therefore,
{*x* : *x* is an even natural number} ⊆ {*x* : *x* is an integer}.

(ii) Since a triangle is not a rectangle, therefore
{*x* : *x* is a triangle in the plane } $\not\subseteq$ {*x* : *x* is a rectangle in the plane}.

(iii) Since an isosceles triangle is not necessarily an equilateral triangle, therefore

{ x : x is an isosceles triangle} $\not\subseteq$ {x : x is an equilateral triangle}.

(iv) Since a is not a set, therefore, $a \not\subseteq \{a, \{b\}, c\}$.

(v) Since {{a}} is a set containing exactly one element {a} and {a} is not an element of the set {a, {b}, c}, therefore, {{a}} $\not\subseteq$ {a, {b}, c}.

Illustration 3

Question: **How many elements are in the set**

$A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

$B = \{x : x$ is even integer and $x < 19\}$

$C = \{x : 0 \le x \le 1$ and x is a rational number$\}$

Solution: The elements of A are φ, {φ}, {φ, {φ}}. So A has three elements.

$B = \{x : x = 0, \pm 2, \pm 4, \pm 6, \ldots$ and $x < 19\} = \{\ldots, -4, -2, 0, 2, 4, 6, \ldots, 18\}$

$\therefore$ B is an infinite set.

C is also infinite set because $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ are all elements of C.

Important formulae/points

- *The order in which the elements of a set are written is immaterial thus the set {1, 2,3} and {2, 1, 3} are same.*
- *Two sets A and B are equal if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.*
- *The set {0} is not an empty set as it contains one element 0.*
- *The set {ϕ} is not an empty set as it contain one element ϕ.*
- $A \subseteq B \Rightarrow P(A) \subseteq P(B)$
- *If A has n elements then P(A) has 2^n elements.*

PRACTICE PROBLEMS

PP1. Which of the following collections are sets? Justify your answer.
(i) The collection of all girls in your class. (ii) The collection of all even integers.
(iii) The collection of beautiful girls of the world.

PP2. Write the following sets in set-builder form:
(i) $A = \{1, 4, 9, 16, \ldots\}$ (ii) $A = \{5, 9, 13, 17, 21, \ldots\}$
(iii) $A = \{14, 21, 28, 35, 42, \ldots, 98\}$ (iv) $A = \{3, 6, 9, 12, \ldots\}$

PP3. Write the following sets in tabular form (roster form) :
(i) $A = \{x : x$ is a natural number and $5 < x \le 26\}$
(ii) $B = \{x : x^2 + x - 4 = 0\}$
(iii) $C = \{x : x = 2n - 1 < 20$ and $n \in N\}$
(iv) $D = \left\{x : x = \frac{n}{n+1}, n \in N\right\}$

PP4. Match each of the sets on the left described in the roaster form with the same set on the right described in the set-builder form:
(i) {H, A, R, Y, N } (a) $\{x : x$ is a natural number and a divisor of 18}
(ii) {L, I, T, E} (b) $\{x : x$ is a letter of the word HARYANA}
(iii) {1, 2, 3, 4, 6, 12} (c) $\{x : x$ is a letter of the word LITTLE }
(iv) {1, 2, 3, 6, 9, 18} (d) $\{x : x$ is a natural number and is a divisor of 12}

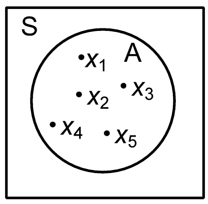
PP5. Determine the empty sets and, singleton set in the following sets:
(i) $A = \{x : x^2 = 2, x$ is a rational number} (ii) $B = \{x : x > 0$ and $x^2 = 25\}$
(iii) $C = \{x : x^3 + 1 = 0$ and x is an integer$\}$

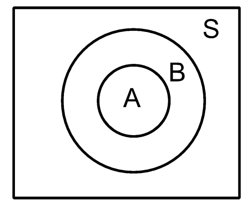
PP6. Find the number of subsets that can be formed from the set $A = \{4, 5, 6\}$.

PP7. If $A = \{1, 2, \{2, 3\}, 4\}$, state which of the following statements are true?
(i) $\{2, 3\} \subset A$ (ii) $\{2, 3\} \in A$
(iii) $\{2, 4\} \in A$ (iv) $\{\{2, 3\}\} \subset A$ (v) $\{1, 2, 3\} \subset A$

4. VENN DIAGRAMS

Statements involving sets can be easily understood with pictorial representation of the sets. A set is represented by circle or a closed geometrical figure A, inside the universal set *S,* which is represented by a rectangular region. Elements of a set *A* are represented by points within the circle which represents *A*.





$A \subseteq B$

4.1 OPERATION ON SETS

In algebra of numbers, the operation of addition (+) when applied on two numbers gives a third number $a+b$. Likewise we discuss the operation union ($\cup$), intersection ($\cap$) and difference (–) applicable on any two sets.

- **Union of two sets**

The union of two sets *A* and *B* is the set of all those elements which are either in *A* or in *B* or in both. This set is denoted by $A \cup B$ (read as '*A* union *B*').

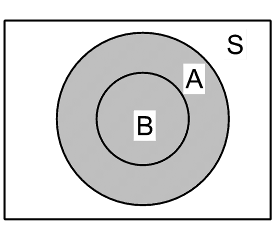
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$= \{x : x \in A \vee x \in B\}$$ { $\vee$ denotes 'or'}

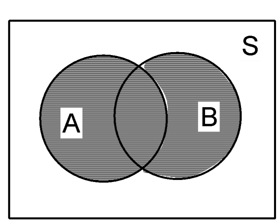
Also, $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 5, 6\}$, then $A \cup B = \{1, 2, 3, 5, 6\}$.

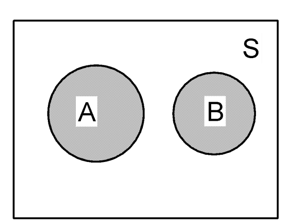
The union of two sets can be represented by Venn diagram as shown in the figure below:



$A \cup B$ when $A \subseteq B$



$A \cup B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A \cup B$
when *A* and *B* disjoint sets

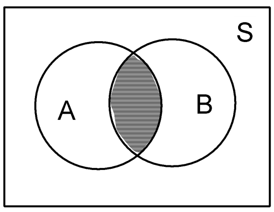
Here shaded portions are $A \cup B$.

- **Intersection of two sets**

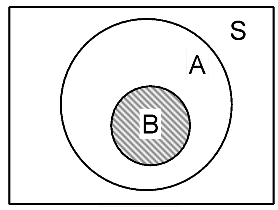
The intersection of two sets *A* and *B* is the set of all the elements which are common in *A* and *B*. This set is denoted as $A \cap B$ and read as *A* intersection *B*.

$$A \cap B = \{x : x \in A \text{ and } x \in B\} = \{x : x \in A \wedge x \in B\}$$ (^ denotes and)

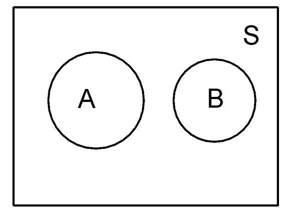
$$x \in A \cap B \Leftrightarrow x \in A \text{ and } x \in B$$



$A \cap B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A \cap B$ when $B \subseteq A$, $A \cap B = B$



$A \cap B = \phi$ no shaded region when *A* and *B* are disjoint sets

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 5, 6\}$, then $A \cap B = \{1, 2\}$

Now, $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

5. DIFFERENCE AND COMPLEMENTS

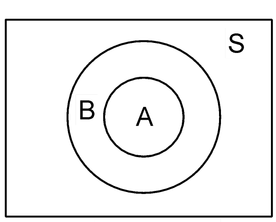
5.1 DIFFERENCE OF TWO SETS

The difference of two sets *A* and *B* is the set of all those elements of *A* which are not elements of *B*. It is denoted by $A - B$.

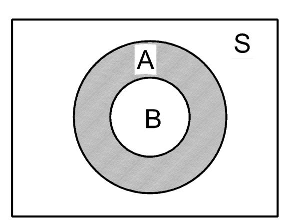
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

thus $$x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$$

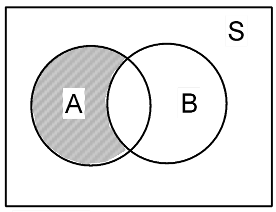
$A - B$ can be represented by Venn diagram (shaded region) as below:



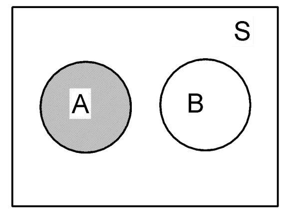
$A - B = \phi$ when $A \subseteq B$



$A - B$ when $B \subseteq A$



$A - B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A - B = A$
when A and B are disjoint sets

Example: Let $A = \{1, 3, 5, 6, 7\}$; $B = \{2, 3, 4, 5\}$, then $A - B = \{1, 6, 7\}$; $B - A = \{2, 4\}$.

5.2 COMPLEMENT OF A SET

The complement of a set A is a set of all those elements of universal set S which are not elements of A. It is denoted by A^C or A'.

$$A' = S - A$$

6. LAWS OF ALGEBRA OF SETS

(i) Associative Law

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) **Commutative Law**

- $A \cap B = B \cap A$
- $A \cup B = B \cup A$

(iii) **Idempotent Law**

- $A \cap A = A$
- $A \cup A = A$

(iv) **Law of *U***

- $A \cap U = A$
- $A \cup U = U$

(v) **Identity Law**

- $A \cup \phi = A$
- $A \cap \phi = \phi$

(vi) **Distributive Law**

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vii) **De Morgan's Law**

- $(A \cap B)' = A' \cup B'$
- $(A \cup B)' = A' \cap B'$

(viii) **Complement Law**

- $(A \cap A') = \phi$
- $(A \cup A') = U$
- $\phi' = U$
- $U' = \phi$

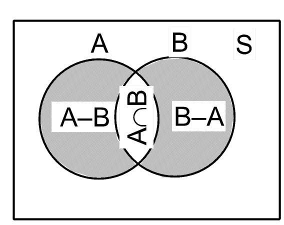
(ix) **Involution Law (Law of double complementation)**

- $(A')' = A$

7. SOME PRACTICAL APPLICATIONS OF SET THEORY

Here we shall study the use of set theory in practical problems.

The number of distinct elements of a finite set A is denoted by $n(A)$.



Use the following results whichever is required

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(iii) $n(A \cup B) = n(A) + n(B - A)$

(iv) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A \cap B = \phi$

(v) $n(A) = n(A - B) + n(A \cap B)$

(vi) $n(B) = n(B - A) + n(A \cap B)$

(vii) Number of elements belonging to exactly one of *A* and *B*

$= n(A - B) + n(B - A)$

$= n(A \cup B) - n(A \cap B)$ $= n(A) + n(B) - 2n(A \cap B)$

(viii) Number of elements belonging to exactly two of *A, B* and *C*

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(x) Number of elements belonging to exactly one of *A, B* and *C*

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(xi) $n(A' \cap B') = n(S) - n(A \cup B)$ $= n(A \cup B)'$

(xii) $n(A' \cup B') = n(S) - n(A \cap B)$

Illustration 4

Question: **If *A* = {1, 3, 5, 6, 7}, *B* = {2, 3, 6, 8} and *C* = {1, 2, 3, 4}, then find**

(i) *A* ∩ *B* **(ii) *A* ∪ *B*** **(iii) *A* – *B*** **(iv) *B* – *A***

Solution: (i) $A \cap B = \{x : x \in A \text{ and } x \in B\} = \{3, 6\}$

(ii) $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{1, 2, 3, 5, 6, 7, 8\}$

(iii) $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 5, 7\}$

(iv) $B - A = \{x : x \in B \text{ and } x \notin A\} = \{2, 8\}$

Illustration 5

Question: **If universal set $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$, $C = \{2, 3, 7\}$, then find (i) A' (ii) $(A - B)'$ (iii) $B' - A'$ (iv) $A' \cap B$ (v) $A \cup B'$ (vi) $(A \cap C)'$.**

Solution: (i) $A' = \{x : x \in S \text{ and } x \notin A\} = \{0, 5, 6, 7, 8, 9\}$

(ii) $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 4\}$

$\therefore \quad (A - B)' = S - (A - B) = S - \{1, 4\} = \{0, 2, 3, 5, 6, 7, 8, 9\}$

(iii) $B' = S - B = S - \{2, 3, 5, 6\} = \{0, 1, 4, 7, 8, 9\}$

$A' = S - A = S - \{1, 2, 3, 4\} = \{0, 5, 6, 7, 8, 9\}$

$B' - A' = \{0, 1, 4, 7, 8, 9\} - \{0, 5, 6, 7, 8, 9\} = \{1, 4\}$.

(iv) $A' \cap B = \{0, 5, 6, 7, 8, 9\} \cap \{2, 3, 5, 6\} = \{5, 6\}$

(v) $A \cup B' = \{1, 2, 3, 4\} \cup \{0, 1, 4, 7, 8, 9\} = \{0, 1, 2, 3, 4, 7, 8, 9\}$

(vi) $A \cap C = \{1, 2, 3, 4\} \cap \{2, 3, 7\} = \{2, 3\}$

$\therefore \quad (A \cap C)' = S - (A \cap C) = \{0, 1, 4, 5, 6, 7, 8, 9\}$

Illustration 6

Question: **If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 8\}$, $C = \{2, 3, 4, 5, 6, 7\}$. Then verify that**

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (ii) $(A \cup B)' = A' \cap B'$

Solution:

(i) $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A \cap B = \{1, 8\}$, $A \cap C = \{2, 4, 6\}$

Now, $A \cap (B \cup C) = \{x : x \in A \text{ and } x \in B \cup C\} = \{1, 2, 4, 6, 8\}$

$(A \cap B) \cup (A \cap C) = \{1, 2, 4, 6, 8\}$

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{x : x \in S \text{ and } x \notin A \cup B\} = \{9\}$

$A' = \{x : x \in S \text{ and } x \notin A\} = \{3, 5, 7, 9\}$

$B' = \{x : x \in S \text{ and } x \notin B\} = \{2, 4, 6, 9\}$

$A' \cap B' = \{9\}$

$\therefore \quad (A \cup B)' = A' \cap B'$.

Illustration 7

Question: **For any three sets *A*, *B*, *C*, prove the following (by using different laws on operations of sets):**

(i) $A - B = B' - A'$ **(ii)** $(A \cup B) \cap (A \cup B') = A$

(iii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Solution:

(i) $A - B = (A \cap B') = B' \cap A = B' \cap (A')' = B' - A'$.

(ii) $(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$ [By distributive law]

$= A \cup \phi = A$

(iii) $(A - B) \cup (B - A) \cup (A \cap B) = [(A \cup B) - (A \cap B)] \cup (A \cap B)]$

$= [(A \cup B) \cap (A \cap B)'] \cup (A \cap B)$

$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$ [By distributive law]

$= (A \cup B) \cap S$, where *S* is the universal set

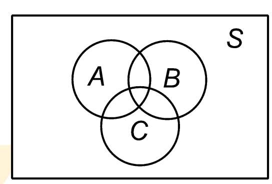
$= A \cup B$.

Illustration 8

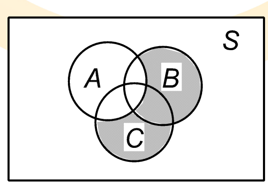
Question: **In the given figure shade the following sets:**

(i) $A' \cap (B \cup C)$

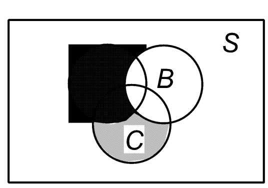
(ii) $A' \cap (C - B)$



Solution: (i) Shaded region represents $A' \cap (B \cup C)$.



(ii) Shaded region represents $A' \cap (C - B)$



Important formulae/points

- $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$
- If $A \subseteq B$, then $A \cup B = B$
- $x \in A^C \Leftrightarrow x \in S$ and $x \notin A$
- $n(A) = n(A - B) + n(A \cap B)$
- $n(B) = n(B - A) + n(A \cap B)$
- $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
- $n(A' \cap B') = n(S) - n(A \cup B) = n(A \cup B)'$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(B) = n(B - A) + n(A \cap B)$
- $n(A \cup B) = n(A) + n(B - A)$

PRACTICE PROBLEMS

PP8. For any three sets *A*, *B*, *C*, prove the following (by using different laws on operations of sets):

(i) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ (ii) $(A \cup B) - C = (A - C) \cup (B - C)$

PP9. Show that the following conditions are equivalent:

(i) $A \subseteq B$ (ii) $A \cap B = A$ (iii) $A - B = \phi$ (iv) $A \cup B = B$

PP10. If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 8\}$, $C = \{2, 3, 4, 5, 6, 7\}$. Then verify that

(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A - C = A \cap C'$

PP11. If 63% of persons like orange where 76% like apples, what can be said about the % of persons who like both oranges and apples?

PP12. In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the number of students who have taken both mathematics and biology and the number of those who have taken biology but not mathematics. Each student has taken either mathematics or biology or both.

8. CARTESIAN PRODUCT OF SETS

Let *a* be an arbitrary element of a given set *A* i.e. $a \in A$ and *b* be an arbitrary element of *B* i.e. $b \in B$. Then the pair (*a*, *b*) is an ordered pair. Obviously $(a, b) \neq (b, a)$. The Cartesian product of two sets *A* and *B* is defined as the set of ordered pairs (*a*, *b*). The Cartesian product is denoted $A \times B$.

$$\Rightarrow \quad A \times B = \{(a, b);\ a \in A,\ b \in B\}.$$

In general $A \times B \neq B \times A$ and if *A* or *B* is a null set, then $A \times B = \phi$.

Moreover, $n(A \times B) = n(A) \cdot n(B)$.

Note :

(i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(iii) $A\times(B-C)=(A\times B)-(A\times C)$

(iv) $(A-B)\times C=(A\times C)-(B\times C)$

(v) $(A\cap B)\times C=(A\times C)\cap(B\times C)$

(vi) $(A\cup B)\times C=(A\times C)\cup(B\times C)$

Illustration 9

Question: **If $A = \{2, 5\}$, $B = \{3, 4, 7\}$ and $C = \{3, 4, 8\}$ then evaluate $A\times B$, $B\times A$, $A\times A$ and verify that**

(i) $A\times(B-C)=(A\times B)-(A\times C)$

(ii) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

Solution: Here $A\times B=\{2, 5\}\times\{3, 4, 7\}$ = {(2, 3), (2, 4), (2, 7), (5, 3), (5, 4), (5, 7)}

$B\times A=$ {3, 4, 7} × {2, 5} = {(3, 2), (3, 5), (4, 2), (4, 5), (7, 2), (7, 5)} and

$A\times A$ = {2, 5} × {2, 5} = {(2, 2), (2, 5), (5, 2), (5, 5)}.

Also $A\times C$ = {2, 5} × {3, 4, 8} = {(2, 3), (2, 4), (2, 8), (5, 3), (5, 4), (5, 8)}

$B-C=\{3, 4, 7\}-\{3, 4, 8\}=\{7\}$

⇒ $A\times(B-C)$ = {2, 5} × {7} = {(2, 7) (5, 7)}

$(A\times B)-(A\times C)=$ {(2, 3), (2, 4), (2, 7), (5, 3), (5, 4), (5,7) }

– {(2, 3), (2, 4), (2, 8), (5, 3), (5, 4), (5, 8)} = {(2, 7), 5, 7} = $A\times(B-C)$.

To verify (ii), we write $B\cup C$ = {3, 4, 7, 8}

⇒ $A\times(B\cup C)=\{2, 5\}\times\{3, 4, 7, 8\}$

= {(2, 3), (2, 4), (2, 7), (2, 8), (5, 3), (5, 4), (5, 7), (5, 8)}

and $(A\times B)\cup(A\times C)=$ {(2, 3), (2, 4), (2, 7), (2, 8), (5, 3), (5, 4), (5, 7), (5, 8)}

= $A\times(B\cup C)$

Important formulae/points

- *The Cartesian product is denoted A × B* $\Rightarrow A \times B = \{(a, b);\ a \in A,\ b \in B\}$.
- *The elements of A × B are also called 2-tuples.*
- *If* $A = \phi$ *or* $B = \phi$ *i.e. if at least one of A and B is an empty set, then* $A \times B = \phi$.
- $A \times B \neq \phi \Leftrightarrow A \neq \phi$ and $B \neq \phi$
- *A × B may or may not be equal to B × A.*
- $A \times B = B \times A$ if and only if $A = B$.