



OSCILLATIONS

Simple Harmonic Motion

Syllabus:-Unit X. Oscillations

Periodic motion-period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M) and its equation; Oscillations of a spring-restoring force and force constant; energy in S.H.M-kinetic and potential energies; simple pendulum-derivation of expression for its time period; free, forced and damped oscillations (qualitative ideas only), resonance.

OSCILLATIONS

Periodic Motions

*A motion which repeats itself after equal intervals of time is called periodic motion. If a body moves to and fro repeatedly about a mean position it is called **oscillatory motion**.*

Displacement

There is a measurable property which changes with time during the periodic motion. This physical quantity is called displacement.

Amplitude

The maximum value of the displacement is called amplitude. It can be positive or negative,

Period and Frequency.

The smallest interval of time after which the process repeats itself is called period. In the case of rotational motion time taken for one complete rotation is called period.

The number of periodic motions (vibration or oscillations) made in one second is called frequency ν (n).

Period T is measured in second. S.I. unit of frequency is Hertz.

$$\text{Frequency} = \frac{1}{\text{period}}$$

$$\nu = \frac{1}{T}$$

Angular Frequency (ω)

Rate of change of angular displacement is called angular velocity or angular frequency. It is the angle described by the rotating body in one second.

$$\omega = \frac{\theta}{t}$$

Suppose in t second the angle described is θ radian. Then

When the body completes one rotation the angle described is 360° or 2π radian and the

time taken is the period T . So angular velocity $\omega = \frac{2\pi}{T}$



$$\text{So, } \omega = \frac{2\pi}{T} = 2\pi \nu$$

Definition of SHM

A particle is said to execute simple harmonic motion (shm) if it moves to and fro about a fixed point under the action of restoring force which is directly proportional to the displacement from the fixed point and is always directed towards the fixed points.

This fixed point is called mean position or equilibrium position. It is called mean position because it lies in the middle of the line of oscillation.

Let the displacement of the particle from the mean position be y and F be the force acting on the particle.

Then $F \propto -y$

$$F = -ky$$

-ve sign shows that F and y are oppositely directed. K is called the force constant or spring constant. K is called force constant because it is a constant connecting force with displacement or k is the restoring force per unit displacement. Its unit is Nm^{-1} .

If the motion takes place under a restoring force it is called SHM. If motion takes place under a restoring torque it is called angular SHM.

Differential Equation of Simple Harmonic Motion

From the definition of simple harmonic motion (SHM)

$$F \propto -y$$

$$F = -k y$$

Where F is the restoring force acting on a body of mass executing SHM along y -axis, y is its displacement from the equilibrium position at any instant. K is called the force constant.

$$\text{But } F = ma = m \frac{d^2 y}{dt^2}$$

$$m \frac{d^2 y}{dt^2} = -ky$$

$$\frac{d^2 y}{dt^2} = -\frac{k}{m} y$$

$$\frac{d^2 y}{dt^2} + \omega^2 y = 0 \quad \text{where } \omega^2 = \frac{k}{m}$$

This equation is the differential equation of SHM. ω is the angular frequency. But

$$\omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$



$$T = 2\pi\sqrt{\frac{m}{k}},$$

Period of oscillation

Where **m** is called the inertia factor and **k** is called the spring factor.

$$= \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{\text{spring factor}}{\text{Inertia factor}}}$$

Frequency of oscillation ν

The solution of this equation can be written as

$$y = a \sin(\omega t + \phi_0)$$

Uniform Circular Motion and simple Harmonic Motion

Consider a particle P moving along the circumference of the circle of radius 'a' with a uniform angular velocity ω . Q is the foot of the perpendicular drawn from P to the diameter YOY'. Q is called the projection of P on the diameter. As the particle moves along the circumference, Q moves to and fro along the diameter about the centre O. The motion of Q about O is said to be simple harmonic. O is called the mean position or equilibrium position. The time taken by Q for moving from Y to Y' and back Y is called the **time period T** and it is equal to the period of rotation of the particle P

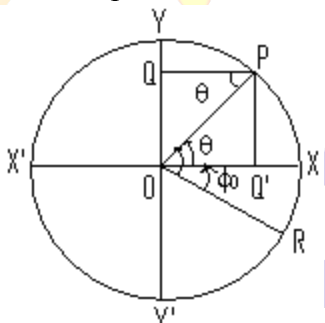


Fig.- 1

SHM is also defined as projection of uniform circular motion upon a diameter of the circle. This circle is called the circle of reference and Q is called the vibrating particle.

Displacement

Let the particle start from X at time $t = 0$ and reach P after t s. Let $\angle XOP = \theta$. The distance OQ measured from O is the displacement of the vibrating particle, $OQ = y$

$$\frac{OQ}{OP}$$

From the triangle OQP, $\sin\theta = \frac{OQ}{OP}$

$$OQ = OP \sin\theta$$

Here, $OP = a$; $\theta = \omega t$; $OQ = y$

$$y = a \sin \omega t \quad \text{But } \omega = \frac{2\pi}{T}$$

So,

$$y = a \sin \frac{2\pi}{T} t$$

Suppose the particle started from R when $t = 0$



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Then $\angle ROP = \theta + \phi_0 = \omega t$ and $\theta = \omega t - \phi_0$

$y = a \sin(\omega t - \phi_0)$, where ϕ_0 is called the initial phase or epoch.

If the perpendicular was drawn to the X-axis then the equation for the displacement would have been,

$$OQ' = a \cos(\omega t - \phi_0) ;$$

$$x = a \cos(\omega t - \phi_0)$$

a is the maximum value of the displacement called the amplitude.

The point Y and Y' are the extreme positions of the vibrating point Q.

Expression for Velocity of a Particle Executing SHM

Let us assume that the initial phase ϕ_0 is zero. The displacement of the foot of the perpendicular drawn from P to the diameter Y'OY is

$$y = a \sin \omega t$$

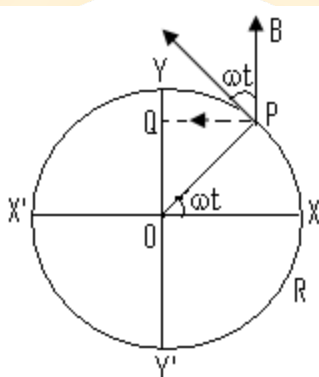


fig -2

Let the velocity of the particle moving around the circle be V . The component of this velocity along PB is $V \cos \omega t$, which is the velocity of Q.

Thus the velocity of the position executing SHM $= V \cos \omega t$

But $V = a\omega$

$$v = a\omega \cos \omega t$$



$v = a\omega \cos \omega t$ $\sin \omega t = \frac{y}{a}$ $\cos \omega t = \sqrt{1 - \sin^2 \omega t}$ $= \sqrt{1 - \frac{y^2}{a^2}}$ $= \sqrt{\frac{a^2 - y^2}{a^2}}$ $v = a\omega \sqrt{\frac{a^2 - y^2}{a^2}}$ $v = +\omega \sqrt{a^2 - y^2},$	<p style="text-align: center;"><u>OR</u></p> $y = a \sin \omega t$ $v = \frac{dy}{dt} = \frac{d(a \sin \omega t)}{dt} = a \cos \omega t \cdot \omega = a\omega \cos \omega t$ $= a\omega \sqrt{1 - \sin^2 \omega t}$ $= \sqrt{1 - \frac{y^2}{a^2}} \quad \text{Since } \sin \omega t = \frac{y}{a}$ $= \sqrt{\frac{a^2 - y^2}{a^2}}$ $v = a\omega \sqrt{\frac{a^2 - y^2}{a^2}}$ $v = +\omega \sqrt{a^2 - y^2},$
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which is the expression for the velocity of a particle executing SHM

When $y = 0$, the particle is at the mean position O, the velocity v is maximum

$$v_{\max} = \omega \sqrt{a^2 - 0}$$

$$v_{\max} = a\omega$$

At the extreme position $y = \pm a$ or $y^2 = a^2$

$$v = \omega \sqrt{a^2 - a^2} = 0$$

Expression for Acceleration

(iii) Acceleration of a particle executing SHM at any instant is,

$$\alpha = \frac{d^2 y}{dt^2} = \frac{d^2 (a \sin \omega t)}{dt^2} = d\left(\frac{d(a \sin \omega t)}{dt}\right) = \frac{d(a\omega \cos \omega t)}{dt} = -a\omega^2 \sin \omega t = -\omega^2 y$$

At the equilibrium position $y = 0$, $\alpha = 0$

At the extreme positions, $y = \pm a$, $\alpha = \pm \omega^2 a$

The magnitude of acceleration is zero at the equilibrium position and maximum at the extreme positions.

At the extreme position the acceleration is maximum and its magnitude is $\alpha_{\max} = \omega^2 a$

$$\omega^2 = \frac{\alpha}{a} = \frac{\text{acceleration}}{\text{displacement}}$$

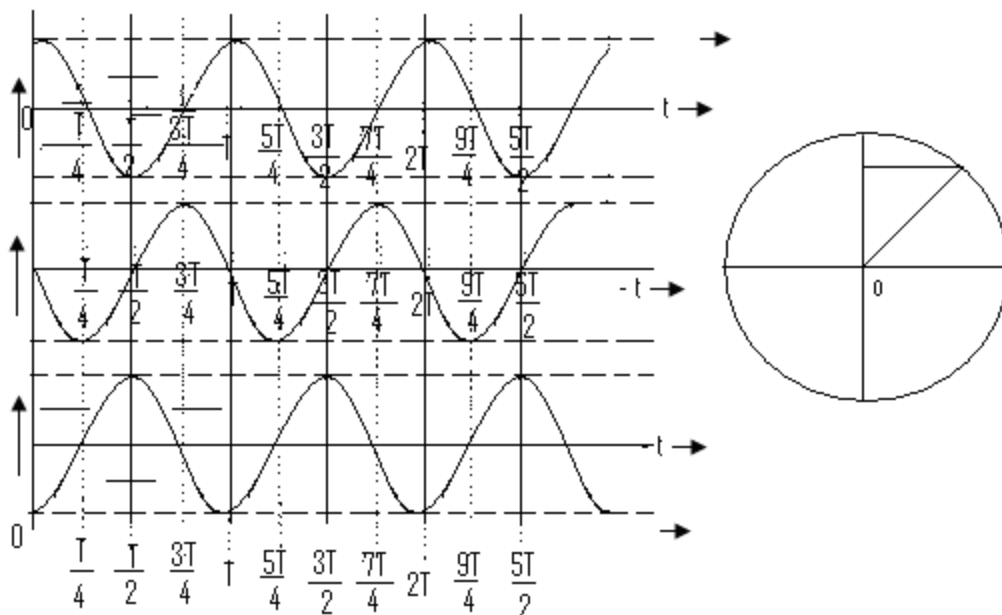
$$\text{Period } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{\text{acceleration}}{\text{displacement}}}}$$



$$T = \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}} = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Graphs to Show the Variation of Displacement, Velocity and Acceleration

At $t = 0$, the vibrating particle is at the extreme position y . So the displacement is maximum, i.e., $x = +a$. At this point Y the velocity is zero and the acceleration is maximum. At $t = T/4$, the particle is at Y' , the displacement is maximum in opposite



direction, i.e., $x = -a$ velocity is zero and acceleration is maximum in the opposite direction. At $t = 3T/4$ the vibrating particle Q reaches the mean position O and at $t = T$, it completes one to and fro motion. Graphs showing the variation of displacement, velocity and acceleration with time is shown in fig.

Phase

Consider a particle executing simple harmonic along the Y -axis. The displacement at any instant t is $y = a \sin(\omega t + \phi_0)$. The argument of the sine function $(\omega t + \phi_0)$ is called phase. ϕ_0 is called the initial phase or epoch.

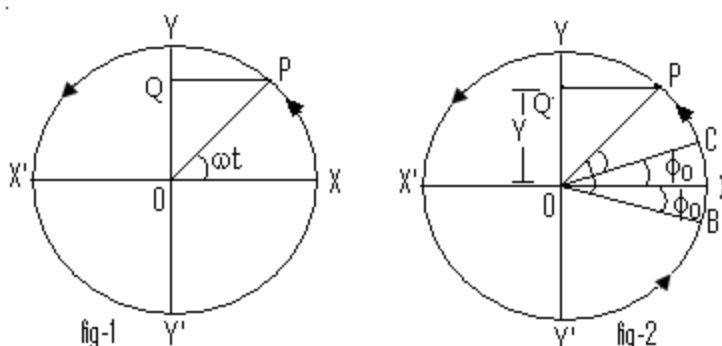
The phase of a particle executing SHM represents the state of position and direction of motion of the particle at any instant. The phase is expressed by angle which the reference particle has subtended at the centre of the reference circle, while passing from the equilibrium position to the position at any instant. The phase is also expressed in terms of the time elapsed since the particle crossed its equilibrium position, by the positive direction.

Suppose we start counting the time when the reference particle is at the point X . After a time t the particle is at P and the angle subtended at the centre by the reference particle is $\angle POX = \omega t$. This angle gives phase of Q , which executes SHM, at any instant t . (fig-1).



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Suppose we start counting time when the particle is at B. Let the particle reach P after a time t . Fig(2)



Then $\angle BOP = \omega t$
 $\angle XOP = \omega t - \phi_0$

The displacement of the particle P is $y = a \sin(\omega t - \phi_0)$.

Suppose we start counting time when the reference particle is at C, i.e, when $t = 0$ the particle is at C. It reaches P after a time t .

Then $\angle COP = \omega t$
 $\angle XOP = \omega t + \phi_0$

The displacement of the particle P is $y = a \sin(\omega t + \phi_0)$

In general $y = a \sin(\omega t \pm \phi_0)$

In this equation ϕ_0 is called the initial phase or epoch

The phase of the particle = $\omega t \pm \phi_0$

$$\phi_0 = \frac{2\pi}{T}t + \phi_0$$

When $t = 0$, $\phi_0 = 0 \pm \phi_0$

Hence the initial phase of the particle is the phase of the particle at time $t = 0$

The rate of change of phase with time is

$$\frac{\Delta\phi}{\Delta t} = \omega$$

This means angular velocity of a particle is equal to the rate of change of phase of the oscillation particle with time.

$$\Delta\phi_0 = \omega\Delta t = \frac{2\pi}{T}\Delta t$$

Suppose the change in time $\Delta t = T$, the period of oscillation.



$$\Delta\phi_0 = \frac{2\pi}{T} \times T = 2\pi$$

Then . This means when the particle completes one to and fro motion or one oscillation the change in phase is 2π , in terms of angle or T , in terms of period. The phase is π or $T/2$ for half oscillation and $\pi/2$ and $T/4$ for a quarter oscillation.

Energy of a harmonic Oscillator

A harmonic oscillator executes SHM under the action of a restoring force. This force always opposes the displacement of the particle. So to displace the particle against this force work must be done. This work done is stored in the particle as its potential energy (P.E.). Since the particle is in motion it has kinetic energy (K.E.). The sum of P.E. and K.E. is always a constant, provided part of this energy, is not used to overcome frictional resistance etc.

Expression for potential Energy

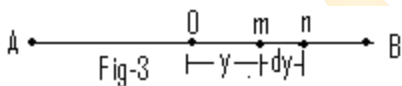
Consider a particle of mass m executing SHM along a line AOB with O as the equilibrium position and A & B as the extreme positions. The force acting on the particle is $F = -ky$ where y is the displacement of the particle from its equilibrium and k is the force constant $k = m\omega^2$, ω is the angular frequency.

Work done to displace the particle by dy = Applied force $\times dy$

$$\text{i.e., } dW = -ky \cdot dy = -kydy \cos 180^\circ = kydy$$

Total work done in displacing the particle by 'y' from its equilibrium position is

$$W = \int_0^y ky dy = k \left[\frac{y^2}{2} \right]_0^y = \frac{1}{2} ky^2$$



$$U = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2$$

$$\text{At equilibrium position } y = 0, \quad U = \frac{1}{2} kx0^2 = 0$$

$$\text{At the extreme position } y = a, \quad U = \frac{1}{2} k a^2 \text{ and is maximum}$$

$$U_{\max} = \frac{1}{2} ka^2 = \frac{1}{2} m\omega^2 a^2$$

Expression for Kinetic Energy

The particle is oscillating. So it has K.E due to its motion denoted by T . The velocity of the particle at a distance y from its equilibrium position is $v = \omega\sqrt{a^2 - y^2}$



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$$T = \frac{1}{2}mv^2, T = \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}k(a^2 - y^2)$$

$$y = 0, \quad K.E = \frac{1}{2}m\omega^2(a^2 - 0). \quad T \text{ is maximum at the mean position}$$

$$T_{\max} = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}ka^2$$

$$\text{At } y = a, \quad T = \frac{1}{2}m\omega^2(a^2 - a^2) = 0, \quad \text{at the extreme position.}$$

Total Energy

At a displacement y , the total energy $E = K.E. + P.E. = T + U$

$$E = \frac{1}{2}k(a^2 - y^2) + \frac{1}{2}ky^2 = \frac{1}{2}ka^2 - \frac{1}{2}ky^2 + \frac{1}{2}ky^2$$

$$E = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2 = T_{\max} = U_{\max}$$

Here m, a, ω are constant
so total energy is constant.

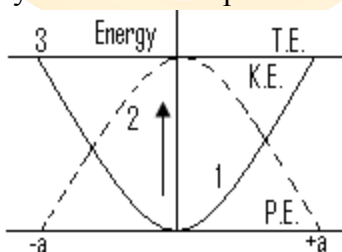
The total energy does not depend on displacement y . It is a constant at all position and at all times.

$$E = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2$$

Total energy can be written as

Graph to Show the Variation of K.E., P.E. and T.E. with Displacement

Graph No. (1) shows the variation of P.E. with displacement. It is parabola. Graph No. (2) shows the variation of K.E. with displacement. It is an inverted parabola. Both K.E. and P.E. terms contain y^2 . That is why the curves are parabolic in shape.



Graph No. (3) is straight line parallel to the X-axis as the total energy is a constant and does not depend on the displacement y .

As we move away from the equilibrium position towards the extreme position, the P.E. increases from zero and K.E. decreases by an equal amount. At the extreme position K.E. is zero and P.E. is maximum.

i.e., At $y = \pm a$, the total energy is in potential form.

At $y = 0$, the total energy is in kinetic form.

Always total energy remains constant.



Characteristics of SHM

- (i) The motion of the body executing SHM is periodic and 'to and fro'
- (ii) The body executing SHM is acted upon by a force which is directly proportional to its displacement from the mean position and directed towards it.
- (iii) The velocity of the body when it is at distance y from the equilibrium position is $v = \omega \sqrt{a^2 - y^2}$. The acceleration is zero at the equilibrium position and maximum at the extreme position.

(iv) The acceleration of the body is $a = -\omega^2 y$. The acceleration is zero at the equilibrium position and maximum at the extreme position.

The total energy is always a constant.

(v) At the mean position the K.E. is maximum and P.E. is zero. At the extreme position K.E. is zero and P.E. is maximum. The total energy is always a constant.

Definition of Spring Constant or Force Constant

The force constant of a spring is defined as the restoring force per unit displacement of the spring.

$$k = \frac{F}{x} \quad \text{S.I. unit of } k \text{ is } \text{Nm}^{-1}$$

Dimensional formula of k is $\text{MLT}^{-2} \times \text{L}^{-1} = [\text{ML}^0\text{T}^{-2}]$

The restoring force acts on the spring and the mass m

$$= \frac{\text{Force}}{\text{mass}} = \frac{kx}{m}$$

The acceleration of the mass

$$\frac{K}{m}$$

Acceleration per unit displacement = $\frac{K}{m}$ where $x=1$

$$T = \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}}$$

\therefore Period of oscillation of spring

$$T = 2\pi \sqrt{\frac{m}{k}}$$

m – mass of the body, k = Spring constant

$$\text{Frequency} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Here the spring was horizontal. Now let us see what effect of gravity on the period of oscillation of the system.

Mass on a Spring – Vertical Oscillation

When the mass m is suspended from the vertical spring, the spring elongates by x_0 , due to the weight mg . This elongation produces a restoring force in the spring acting vertically upwards and it is balanced by the weight of the body mg .

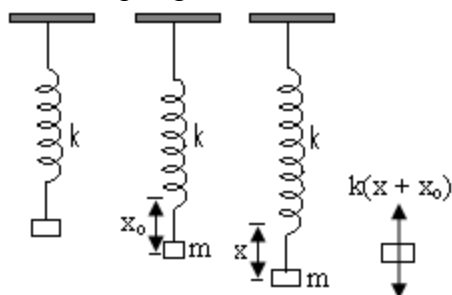
i.e., $mg = kx_0$



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The restoring force = $-kx_o$

The fig. shows the equilibrium and displaced positions of a spring-mass system. At a equilibrium, the deformation in the spring is



$$kx_o = mg$$

$$x_o = \frac{mg}{k}$$

If x be the instantaneous displacement of the block from the equilibrium position, then the net force acting on the block is

$$F = -[k(x + x_o) - mg]$$

$$\text{or } F = -kx$$

Applying Newton's second law

$$m \frac{d^2x}{dt^2} = -kx$$

$$\text{or } \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

The time period is

Note the time period of a spring-mass system is independent of gravity. Gravity only shifts the equilibrium position.

$$\text{Since } kx_o = mg, \text{ therefore, } \frac{m}{k} = \frac{x_o}{g}$$

$$T = 2\pi \sqrt{\frac{x_o}{g}}$$

And thus, where x_o is the deformation at equilibrium.

OR

When the mass m is suspended from the vertical spring, the spring elongates by x_o , due to the weight mg . This elongation produces a restoring force in the spring acting vertically upwards and it is balanced by the weight of the body mg .

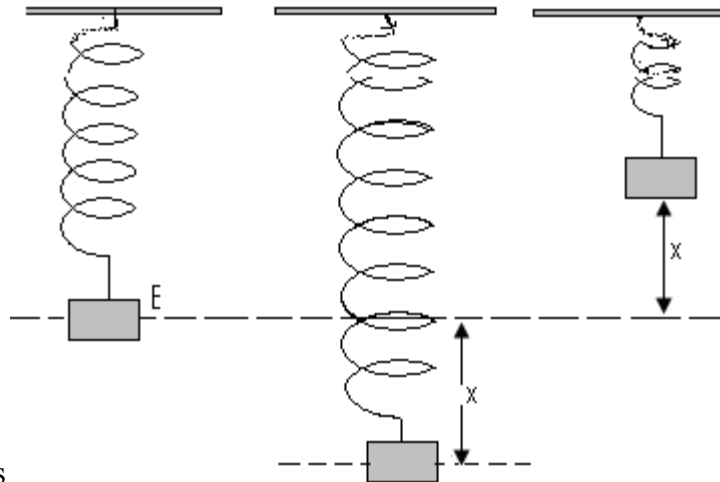
$$\text{i.e., } mg = kx_o$$

$$\text{The restoring force} = -kx_o$$



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This is the equilibrium position E, i.e., the spring and the mass does not move neither upwards nor downwards. From this position suppose the mass m is further pulled down by x . So total restoring force $= k(x + kx_o)$ acting vertically upwards Net force acting



vertically upwards

$$\begin{aligned} &= k(x + kx_o) - mg \\ &= kx + kx_o - mg \\ &= kx, \text{ because } mg = kx_o \end{aligned}$$

This force to bring the mass to its equilibrium position. So it is the restoring force F .

$F = -kx$, negative sign shows that x and F are oppositely directed. The same acts if the spring is compressed by x ,

$$\text{Acceleration} = \frac{-k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Period of oscillation

The equation for period is same, whether the spring is horizontal or vertical. Thus the period is not affected by gravity. Frequency of oscillation.

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\text{From equation (i)} \quad \frac{k}{m} = \frac{g}{x_o}$$

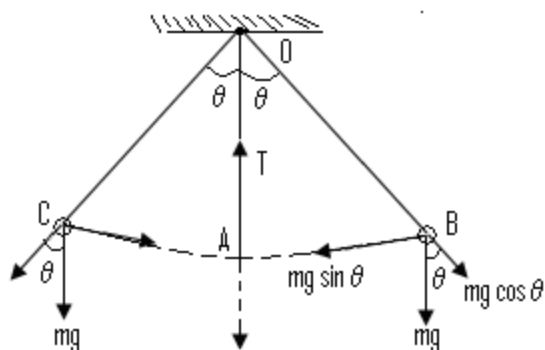


So
$$n = \frac{1}{2\pi} \sqrt{\frac{g}{x_0}}$$

Simple Pendulum

A heavy body suspended by a light inextensible thread is called simple pendulum. A small metal sphere is suspended from a rigid support using a cotton or silk thread. The point from which the bob is suspended is called the point of suspension. The centre of the body is called centre of oscillations. The distance between point of suspension and the centre of oscillation is called length of pendulum.

At A, the weight of the bob acts vertically downwards and the tension in the string acts vertically upwards. These two forces are equal and opposite. So A is the equilibrium position.



Let the bob be displaced by a small angle θ from the equilibrium position towards B. The weight mg is resolved into two rectangular components. The component $mg \cos \theta$ acts radially along OB. This force is equal and opposite to the tension in the string. The component $mg \sin \theta$ acts tangentially along BA. This acts towards the equilibrium position. So it is called the restoring force F.

$$F = -mg \sin \theta$$

$$= -mg \theta, \text{ if } \theta \text{ is small and in radian,}$$

From the, Arc AB = $l \theta$

$$x = l \theta$$

$$\theta = \frac{x}{l}$$

Substituting the value of θ in F,

$$F = \frac{-mg}{l} x$$

Restoring force F

$$F = -kx$$

$$k = \frac{mg}{l}$$

Where the force constant

The equation for restoring F shows that the restoring force is directly proportional to the displacement and directed towards the equilibrium position. The force constant

$$k = mg/l$$



$$\frac{mg}{l} = \frac{l}{g}$$

Period of oscillation

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{l}{g}}$$

Hence the period of oscillation is independent of the mass of the bob. Such oscillations or vibrations are called isochronous vibrations. The period T is directly proportional to the square root of the length of the pendulum, at a place.

Seconds Pendulum

A pendulum whose period is equal to two seconds is called seconds pendulum. This means time taken for half oscillations is one second.

Free Oscillations, Natural Vibration

The oscillation made by a body which is not subjected to any external force is called free oscillation or natural oscillation.

Its frequency is called natural frequency $\nu_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ Where m is the mass of the oscillator and k is the force constant.

For example.

- (i) The vibrations of a tuning fork
- (ii) The oscillations of a simple pendulum.

The graph showing the variation of displacement of a body executing free oscillations with time is as shown in fig. The amplitude of the free oscillations will remain constant only for an ideal oscillator. In practice the free oscillations of an oscillator gradually decreases in amplitude and finally dies away due to frictional forces as shown in fig. Such oscillations are called damped oscillations.

An oscillations can be made to oscillate with constant amplitude by applying a suitable external periodic force. Oscillations are called forced oscillations.

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The oscillation made by a body which is not subjected to any external force is called free

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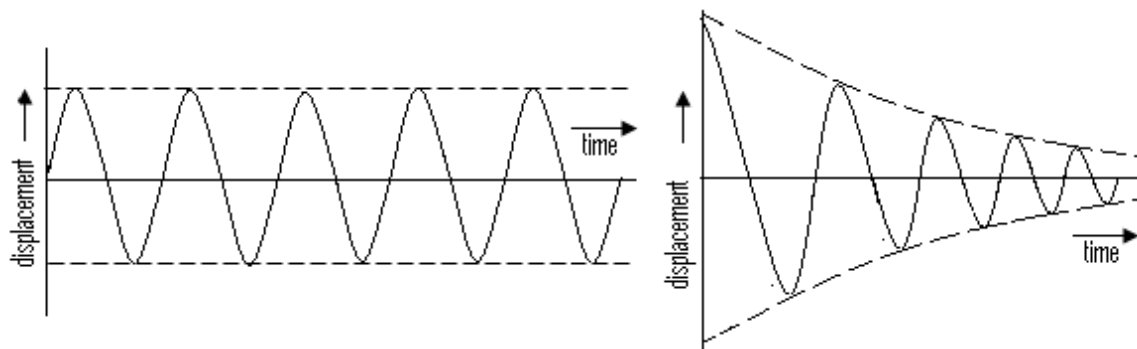
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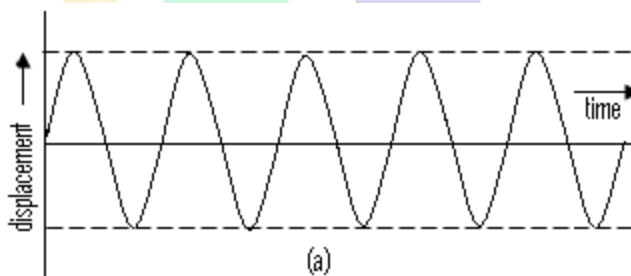


Damped Oscillations

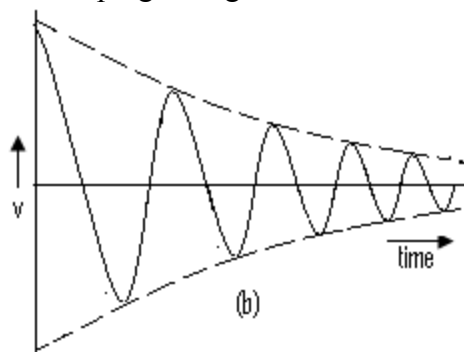
If a vibrating tuning fork is left to itself it is supposed to vibrate indefinitely with a constant amplitude. But in actual practice the amplitude of oscillations gradually decreases to zero due to frictional forces acting on them. Such **periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations.**

Effect of Damping on the Oscillations of Mass Attached to a spring

The variation of displacement of a mass with time is as shown in fig. (a) shows the case when there is no dissipative force. The displacement is alternately positive and negative. The amplitude remains constant.

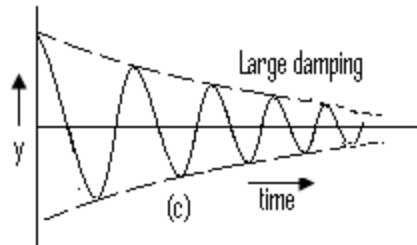


(b) shows the case when there is small damping. Due to the damping force the amplitude gradually decreases when the damping is large the variation is shown in fig.

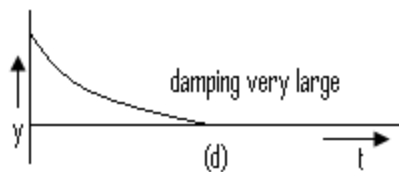




(c) If the damping is very large there is no oscillation.



(d) The displacement to zero as shown in fig.



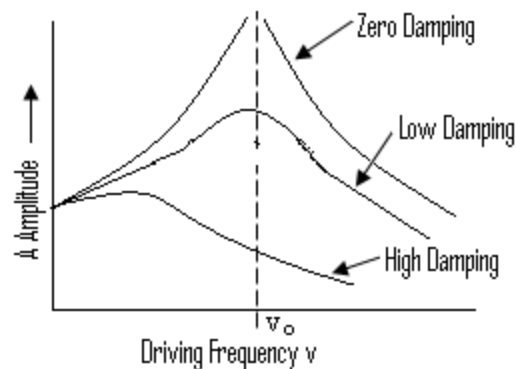
Forced Harmonic Oscillations

Suppose an external periodic force which varies harmonically with time is exerted on an oscillator. At first the body tries to vibrate with its natural frequency, while the applied force tries to drive the body with its frequency. But soon the free vibrations of the body die out and finally the body vibrates with constant amplitude and with the same frequency as that of the applied force.

An oscillator thus compelled to oscillate with a frequency, other than its natural frequency is called forced harmonic oscillator. The oscillations made by the oscillator under the action of the external periodic force with a frequency other than its natural frequency are called forced oscillations.

Resonance

Consider a system whose natural frequency is N vib/s when it oscillates. Suppose the system is at rest. Let a periodic force having a frequency N_1 vib/s, but the system to vibrate with a frequency N vib/s. Suppose the frequency of the driving force is gradually reduced to N vib/s. As the frequency of the driving force becomes close to the natural frequency of the system, its amplitude increases and becomes larger when the frequencies are equal. This phenomenon is called resonance.



At resonance, the amplitude of the oscillator depends upon the dissipative force. The variation of amplitude with frequency is shown in fig. From the figure it can be seen that

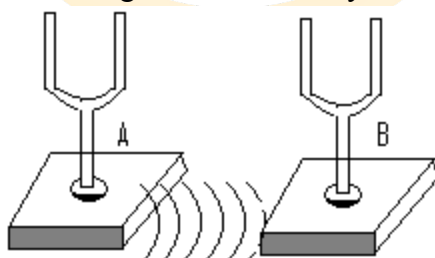


the amplitude is greater for low damping. For zero damping the amplitude becomes infinity at resonance. This amplitude is low at high damping.

Resonance is the phenomenon in which a system (a body) is made to vibrate by an external force (driving force) whose frequency is equal to the natural frequency of the system. At resonance the amplitude of the system is maximum and the energy supplied by the driving force is just sufficient to overcome friction in the system.

Examples to Illustrate Resonance

1. Marching soldiers are ordered to break their steps while crossing a suspension bridge. This is because the frequencies of their steps may become equal to the natural frequency of the bridge. Thus the bridge becomes in resonance with their foot steps and its amplitude becomes maximum. Hence the bridge may collapse.
2. A singer can break a well-made glassware by producing a note of suitable frequency. When the singer sings the energy absorbed by the glass makes it vibrate and for the right note the glass shatters.
3. The rattling produced in car at a certain speed is due to resonance. This happens when the frequency of the sound is equal to the natural frequency of the glass.
4. Distant explosion can break the glass window due to resonance. This happens when the frequency of the sound is equal to the natural frequency of the glass.
5. A child pushing a swing can reach the maximum height if the frequency of push is equal to the natural frequency of the swing.
6. In a rough sea if the natural frequency of a ship is equal to the frequency of the water wave then the amplitude of swinging of the ship may cross the limit of safety and may become dangerous. This is avoided by changing the speed and direction of the ship.
7. By keeping two tuning forks of same frequency close together on a sounding box and if one of them say A is vibrated, then the other B is also seen to vibrate due to resonance. This is because the energy from the vibrating tuning fork A reaches the prong of the other fork B at the right time in each cycle.





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