Units, Dimensions and Measurement

Physical Quantity

A quantity which can be measured by scientific way and which various physical happenings can be explained and expressed in form of laws is called a physical quantity. For example length, mass, time, force etc.

Measurement is necessary to determine magnitude of a physical quantity, to compare two similar physical quantities and to prove physical laws or equations.

A physical quantity is represented completely by its magnitude and unit. For example, 10 *metre* means a length which is ten times the unit of length. Here 10 represents the numerical value of the given quantity and *metre* represents the unit of quantity under consideration. Thus in expressing a physical quantity we choose a unit and then find that how many times that unit is contained in the given physical quantity, *i.e.*

$$\text{Physical quantity } (Q) = \text{Magnitude} \times \text{Unit} = n \times u$$

Where, n represents the numerical value and u represents the unit. Thus while expressing definite amount of physical quantity, it is clear that as the unit(u) changes, the magnitude(n) will also change but product 'nu' will remain same.

i.e. $n\,u$ = constant, or $n_1u_1 = n_2u_2 = \text{constar}$; $\therefore$ $n \propto \frac{1}{u}$

i.e. magnitude of a physical quantity and units are inversely proportional to each other .Larger the unit, smaller will be the magnitude.

Types of Physical Quantity

Ratio (numerical value only) : When a physical quantity is a ratio of two similar quantities, it has no unit.

e.g. Relative density = Density of object/Density of water at 4°C

Refractive index = Velocity of light in air/Velocity of light in medium

Strain = Change in dimension/Original dimension

Note : ❑ Angle is exceptional physical quantity, which though is a ratio of two similar physical quantities (angle = arc / radius) but still requires a unit (degrees or radians) to specify it along with its numerical value.

(2) **Scalar (Magnitude only) :** Those physical quantities which have only magnitude is called scalar. These physical quantities do not have any direction *e.g.* Length, time, work, energy *etc.*

Magnitude of a physical quantity can be negative. In that case negative sign indicates that the numerical value of the quantity under consideration is negative. It does not specify the direction.

Scalar quantities can be added or subtracted with the help of following ordinary laws of addition or subtraction.

(3) **Vector (magnitude and direction) :** Those physical quantities which have magnitude as well as direction are called vector. These physical quantities have a direction *e.g.* displacement, velocity, acceleration, force *etc.*

Vector physical quantities can be added or subtracted according to vector laws of addition.

Note : ❑ There are certain physical quantities which behave neither as scalar nor as vector. For example, moment of inertia is not a vector as by changing the sense of rotation its value is not changed.

It is also not a scalar as it has different values in different directions (*i.e.* about different axes). Such physical quantities are called ***Tensors.*** Other example of Tensors is Electric Current

Fundamental and Derived Quantities

(1) **Fundamental quantities :** Out of large number of physical quantities which exist in nature, there are only few quantities which are independent of all other quantities and do not require the help of any other physical quantity for their definition, therefore these are called absolute quantities. These quantities are also called fundamental or base quantities, as all other quantities are based upon and can be expressed in terms of these quantities.

(2) **Derived quantities :** All other physical quantities can be derived by suitable multiplication or division of different powers of fundamental quantities. These are therefore called derived quantities.
If length is defined as a fundamental quantity then area and volume are derived from length and are expressed in term of length with power 2 and 3 over the term of length.

Note : ❑ In mechanics Length, Mass and time are arbitrarily chosen as fundamental quantities. However this set of fundamental quantities is not a unique choice. In fact any three quantities in mechanics can be termed as fundamental as all other quantities in mechanics can be expressed in terms of these. *e.g.* if speed and time are taken as fundamental quantities, length will become a derived quantity because then length will be expressed as Speed × Time. and if force and acceleration are taken as fundamental quantities, then mass will be defined as Force / acceleration and will be termed as a derived quantity.

Fundamental and Derived Units

Normally each physical quantity requires a unit or standard for its specification so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in *mechanics* we choose arbitrarily units of any *three* physical quantities we can express the units of all other physical quantities in mechanics in terms of these. Arbitrarily the physical quantities *mass*, *length* and *time* are choosen for this purpose. *So any unit of mass, length and time in mechanics is called a* **fundamental, absolute or base unit.** *Other units which can be expressed in terms of fundamental units, are called derived units.* For example light year or *km* is a fundamental units as it is a unit of length while s^{-1}, m^2 or kg/m are derived units as these are derived from units of time, mass and length respectively.

System of units : A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units. The common systems are given below –

(1) **CGS system :** The system is also called Gaussian system of units. In it length, mass and time have been chosen as the fundamental quantities and corresponding fundamental units are centimetre (*cm*), gram (*g*) and second (*s*) respectively.

(2) **MKS system :** The system is also called Giorgi system. In this system also length, mass and time have been taken as fundamental quantities, and the corresponding fundamental units are *metre*, kilogram and second.

(3) **FPS system :** In this system foot, pound and second are used respectively for measurements of length, mass and time. In this system force is a derived quantity with unit poundal.

(4) **S. I. system :** It is known as International system of units, and is infact extended system of units applied to whole physics. There are seven fundamental quantities in this system. These quantities and their units are given in the following table

Quantity	Name of Unit	Symbol
Length	metre	*m*
Mass	kilogram	*kg*
Time	second	*s*
Electric Current	ampere	*A*

Temperature	Kelvin	*K*
Amount of Substance	mole	*mol*
Luminous Intensity	candela	*cd*

Besides the above seven fundamental units two supplementary units are also defined –
Radian (*rad*) for plane angle and Steradian (*sr*) for solid angle.
Note : ❑ Apart from fundamental and derived units we also use very frequently practical units. These may be fundamental or derived units
e.g., light year is a practical unit (fundamental) of distance while horse power is a practical unit (derived) of power.
Practical units may or may not belong to a system but can be expressed in any system of units *e.g.*, 1 mile = 1.6 *km* = 1.6×10^3 *m*.

Standards of Length, Mass and Time

(1) **Length :** Standard metre is defined in terms of wavelength of light and is called atomic standard of length.
The metre is the distance containing 1650763.73 wavelength in vacuum of the radiation corresponding to orange red light emitted by an atom of krypton-86.
Now a days metre is defined as length of the path travelled by light in vacuum in 1/299,7792, 458 part of a second.
(2) **Mass :** The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights
and Measures is defined as 1 *kg*.
On atomic scale, 1 *kilogram* is equivalent to the mass of 5.0188×10^{25} atoms of $_6C^{12}$ (an isotope of carbon).

(3) **Time :** 1 *second* is defined as the time interval of 9192631770 vibrations of radiation in *Cs*-133 atom.
This radiation corresponds to the transition between two hyperfine level of the ground state of *Cs*-133.

Practical Units

(1) **Length :**
(i) 1 fermi = 1 *fm* = 10^{-15} *m*
(ii) 1 *X*-ray unit = 1*XU* = 10^{-13} *m*
(iii) 1 angstrom = 1Å = 10^{-10} *m* = 10^{-8} *cm* = 10^{-7} *mm* = 0.1 *μmm*
(iv) 1 micron = *μm* = 10^{-6} *m*
(v) 1 astronomical unit = 1 *A.U.* = 1.49×10^{11} *m* ≈ 1.5×10^{11} *m* ≈ 10^8 *km*
(vi) 1 Light year = 1 *ly* = 9.46×10^{15} *m*
(vii) 1 *Parsec* = 1*pc* = 3.26 light year
(2) **Mass :**
(i) Chandra Shekhar unit : 1 *CSU* = 1.4 times the mass of sun = 2.8×10^{30} *kg*
(ii) Metric tonne : 1 Metric tonne = 1000 *kg*
(iii) Quintal : 1 Quintal = 100 *kg*
(iv) Atomic mass unit (*amu*) : *amu* = 1.67×10^{-27} *kg* mass of proton or neutron is of the order of 1 *amu*
(3) **Time :**
(i) Year : It is the time taken by earth to complete 1 revolution around the sun in its orbit.
(ii) Lunar month : It is the time taken by moon to complete 1 revolution around the earth in its orbit.
1 *L.M.* = 27.3 days
(iii) Solar day : It is the time taken by earth to complete one rotation about its axis with respect to sun.

Since this time varies from day to day, average solar day is calculated by taking average of the duration of all the days in a year and this is called Average Solar day.

1 Solar year = 365.25 average solar day

or average solar day $= \frac{1}{365.25}$ the part of solar year

(iv) Sedrial day : It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

1 Solar year = 366.25 Sedrial day = 365.25 average solar day

Thus 1 Sedrial day is less than 1 solar day.

(v) Shake : It is an obsolete and practical unit of time.

1 Shake = 10^{-8} *sec*

Dimensions of a Physical Quantity

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force

Force = mass × acceleration $= \frac{\text{mass} \times \text{velocity}}{\text{time}} = \frac{\text{mass} \times \text{length/time}}{\text{time}}$ = mass × length × (time)$^{-2}$ .... (i)

Thus, the dimensions of force are 1 in mass, 1 in length and – 2 in time.

Here the physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to indicate that the equation is among the dimensions and not among the magnitudes.

Thus equation (i) can be written as [force] = $[MLT^{-2}]$.

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the R.H.S. of the equation, the expression is termed as dimensional formula.

Thus, dimensional formula for force is, $[MLT^{-2}]$.

Application of Dimensional Analysis

(1) **To find the unit of a physical quantity in a given system of units** : Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing *M*, *L* and *T* by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, *e.g.*, Work = Force × Displacement

So $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So its units in C.G.S. system will be $g\ cm^2/s^2$ which is called *erg* while in M.K.S. system will be $kg\ m^2/s^2$ which is called *joule*.

Sample problems based on unit finding

Problem **1.** If $x = at + bt^2$, where x is the distance travelled by the body in *kilometre* while t the time in seconds, then the units of b are

(a) *km/s* (b) *km-s* (c) km/s^2 (d) $km\text{-}s^2$

Solution : (c) From the principle of dimensional homogeneity $[x] = [bt^2] \Rightarrow [b] = \left[\frac{x}{t^2}\right]$

∴ Unit of $b = km/s^2$.

Problem **2.** The unit of surface tension in SI system is

(a) $Dyne/cm^2$ (b) *Newton/m* (c) *Dyne/cm* (d) $Newton/m^2$

Solution : (b) From the formula of surface tension $T = \frac{F}{l}$

By substituting the S.I. units of force and length, we will get the unit of surface tension = *Newton/m*

***Problem* 3.** The equation $\left(P + \frac{a}{V^2}\right)(V - b)$ = constant. The units of *a* is

(a) $Dyne \times cm^5$ (b) $Dyne \times cm^4$ (c) $Dyne / cm^3$ (d) $Dyne / cm^2$

Solution : (b) According to the principle of dimensional homogeneity $[P] = \left[\frac{a}{V^2}\right]$

$\Rightarrow [a] = [P][V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$

or unit of a = $gm \times cm^5 \times sec^{-2}$ = Dyne × cm^4

***Problem* 4.** The unit of absolute permittivity is

(a) *Farad - meter* (b) *Farad / meter* (c) *Farad/meter*^{2} (d) *Farad*

Solution : (b) From the formula $C = 4\pi\varepsilon_0 R$ $\therefore \varepsilon_0 = \frac{C}{4\pi R}$

By substituting the unit of capacitance and radius : unit of ε_0 = *Farad/ meter*.

***Problem* 5.** Unit of Stefan's constant is **[MP PMT 1989]**

(a) Js^{-1} (b) $Jm^{-2}s^{-1}K^{-4}$ (c) Jm^{-2} (d) Js

Solution : (b) Stefan's formula $\frac{Q}{At} = \sigma T^4$ $\therefore \sigma = \frac{Q}{AtT^4}$ $\therefore$ Unit of $\sigma = \frac{\text{Joule}}{m^2 \times sec \times K^4} = Jm^{-2}s^{-1}K^{-4}$

***Problem* 6.** The SI unit of universal gas constant (*R*) is

(a) *Watt* $K^{-1}mol^{-1}$ (b) *Newton* $K^{-1}mol^{-1}$ (c) *Joule* $K^{-1}mol^{-1}$ (d) $ErgK^{-1}mol^{-1}$

Solution : (c) Ideal gas equation $PV = nRT$ $\therefore [R] = \frac{[P][V]}{[nT]} = \frac{[ML^{-1}T^{-2}][L^3]}{[mole][K]} = \frac{[ML^2T^{-2}]}{[mole] \times [K]}$

So the unit will be *Joule* $K^{-1}mol^{-1}$.

(2) **To find dimensions of physical constant or coefficients** : As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant : According to Newton's law of gravitation $F = G\frac{m_1 m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2}$

Substituting the dimensions of all physical quantities $[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$

(ii) Plank constant : According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities $[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

(iii) Coefficient of viscosity : According to Poiseuille's formula $\frac{dV}{dt}=\frac{\pi pr^4}{8\eta l}$ or $\eta=\frac{\pi pr^4}{8l(dV/dt)}$

Substituting the dimensions of all physical quantities $[\eta]=\frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]}=[ML^{-1}T^{-1}]$

Sample problems based on dimension finding

<u>Problem</u> **7.** A force *F* is given by $F=at+bt^2$, where t is time. What are the dimensions of *a* and *b*

(a) MLT^{-3} and ML^2T^{-4} (b) MLT^{-3} and MLT^{-4} (c) MLT^{-1} and MLT^0 (d) MLT^{-4} and MLT^1

Solution : (b) From the principle of dimensional homogenity $[F]=[at]$ $\therefore$ $[a]=\left[\frac{F}{t}\right]=\left[\frac{MLT^{-2}}{T}\right]=[MLT^{-3}]$

Similarly $[F]=[bt^2]$ $\therefore$ $[b]=\left[\frac{F}{t^2}\right]=\left[\frac{MLT^{-2}}{T^2}\right]=[MLT^{-4}]$.

<u>Problem</u> **8.** $X=3YZ^2$ find dimension of Y in (MKSA) system, if X and Z are the dimension of capacity and magnetic field respectively

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2} (c) $M^{-3}L^{-2}T^4A^4$ (d) $M^{-3}L^{-2}T^8A^4$

Solution : (d) $X=3YZ^2$ $\therefore$ $[Y]=\frac{[X]}{[Z^2]}=\frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}=[M^{-3}L^{-2}T^8A^4]$.

<u>Problem</u> **9.** Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(a) $[LT^{-1}]$ (b) $[L^{-1}T]$ (c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$

Solution : (d) We know that velocity of light $C=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\therefore$ $\frac{1}{\mu_0\varepsilon_0}=C^2$

$\therefore$ So $\left[\frac{1}{\mu_0\varepsilon_0}\right]=[LT^{-1}]^2=[L^2T^{-2}]$.

<u>Problem</u> **10.** If *L*, *C* and *R* denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is

(a) $[ML^{-2}T^{-1}I^0]$ (b) $[M^0L^0T^3I^0]$ (c) $[M^{-1}L^{-2}T^6I^2]$ (d) $[M^0L^0T^2I^0]$

Solution : (b) $[C^2LR]=\left[C^2L^2\frac{R}{L}\right]=\left[(LC)^2\left(\frac{R}{L}\right)\right]$

and we know that frequency of LC circuits is given by $f=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ *i.e.*, the dimension of LC is equal to $[T^2]$

and $\left[\frac{L}{R}\right]$ gives the time constant of $L-R$ circuit so the dimension of $\frac{L}{R}$ is equal to [*T*].

By substituting the above dimensions in the given formula $\left[(LC)^2\left(\frac{R}{L}\right)\right]=[T^2]^2[T^{-1}]=[T^3]$.

Problem **11.** The position of a particle at time t is given by the relation $x(t)=\left(\frac{v_0}{\alpha}\right)(1-c^{-\alpha t})$, where v_0 is a constant and $\alpha>0$. The dimensions of v_0 and α are respectively **[CBSE 1995]**

(a) $M^0L^1T^{-1}$ and T^{-1} (b) $M^0L^1T^0$ and T^{-1} (c) $M^0L^1T^{-1}$ and LT^{-2} (d) $M^0L^1T^{-1}$ and T

Solution : (a)From the principle of dimensional homogeneity $[\alpha\, t]$ = dimensionless $\therefore$ $[\alpha]=\left[\frac{1}{t}\right]=[T^{-1}]$

Similarly $[x]=\frac{[v_0]}{[\alpha]}$ $\therefore$ $[v_0]=[x][\alpha]=[L][T^{-1}]=[LT^{-1}]$.

Problem **12.** The dimensions of physical quantity X in the equation Force $=\frac{X}{\text{Density}}$ is given by **[DCE 1993]**

(a) $M^1L^4T^{-2}$ (b) $M^2L^{-2}T^{-1}$ (c) $M^2L^{-2}T^{-2}$ (d) $M^1L^{-2}T^{-1}$

Solution : (c) $[X]$ = [Force] × [Density] = $[MLT^{-2}]\times[ML^{-3}]$ = $[M^2L^{-2}T^{-2}]$.

Problem **13.** Number of particles is given by $n=-D\frac{n_2-n_1}{x_2-x_1}$ crossing a unit area perpendicular to X-axis in unit time, where n_1 and n_2 are number of particles per unit volume for the value of x meant to x_2 and x_1. Find dimensions of D called as diffusion constant

(a) M^0LT^2 (b) $M^0L^2T^{-4}$ (c) M^0LT^{-3} (d) $M^0L^2T^{-1}$

Solution : (d)(n) = Number of particle passing from unit area in unit time = $\frac{\text{No. of particle}}{A\times t}=\frac{[M^0L^0T^0]}{[L^2][T]}=[L^{-2}T^{-1}]$

$[n_1]=[n_2]=$ No. of particle in unit volume = $[L^{-3}]$

Now from the given formula $[D]=\frac{[n][x_2-x_1]}{[n_2-n_1]}=\frac{[L^{-2}T^{-1}][L]}{[L^{-3}]}=[L^2T^{-1}]$.

Problem **14.** E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5G^2}$ are **[AIIMS 1985]**

(a)Angle (b)Length (c)Mass (d) Time

Solution : (a) $[E]$ = energy = $[ML^2T^{-2}]$, $[m]$ = mass = $[M]$, $[l]$ = Angular momentum = $[ML^2T^{-1}]$

$[G]$ = Gravitational constant = $[M^{-1}L^3T^{-2}]$

Now substituting dimensions of above quantities in $\frac{El^2}{m^5G^2}=\frac{[ML^2T^{-2}]\times[ML^2T^{-1}]^2}{[M^5]\times[M^{-1}L^3T^{-2}]^2}=[M^0L^0T^0]$

i.e., the quantity should be angle.

Problem **15.** The equation of a wave is given by $Y=A\sin\omega\left(\frac{x}{v}-k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is **[MP PMT 1993]**

(a) LT (b) T (c) T^{-1} (d) T^2

Solution : (b)According to principle of dimensional homogeneity $[k]=\left[\frac{x}{v}\right]=\left[\frac{L}{LT^{-1}}\right]=[T]$.

Problem **16.** The potential energy of a particle varies with distance *x* from a fixed origin as $U=\frac{A\sqrt{x}}{x^2+B}$, where *A* and *B* are dimensional constants then dimensional formula for *AB* is

(a)$ML^{7/2}T^{-2}$ (b) $ML^{11/2}T^{-2}$ (c) $M^2L^{9/2}T^{-2}$ (d) $ML^{13/2}T^{-3}$

Solution : (b)From the dimensional homogeneity $[x^2]=[B]$ $\therefore$ $[B]=[L^2]$

As well as $[U]=\frac{[A][x^{1/2}]}{[x^2]+[B]}$ $\Rightarrow$ $[ML^2T^{-2}]=\frac{[A][L^{1/2}]}{[L^2]}$ $\therefore [A]=[ML^{7/2}T^{-2}]$

Now $[AB]=[ML^{7/2}T^{-2}]\times[L^2]$ $=[ML^{11/2}T^{-2}]$

Problem **17.** The dimensions of $\frac{1}{2}\varepsilon_0E^2$ (ε_0 = permittivity of free space ; *E* = electric field) is **[IIT-JEE 1999]**

(a) MLT^{-1} (b) ML^2T^{-2} (c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

Solution : (c) Energy density = $\frac{1}{2}\varepsilon_0E^2=\frac{\text{Energy}}{\text{Volume}}=\left[\frac{ML^2T^{-2}}{L^3}\right]=[ML^{-1}T^{-2}]$

Problem **18.** You may not know integration. But using dimensional analysis you can check on some results. In the integral $\int\frac{dx}{(2ax-x^2)^{1/2}}=a^n\sin^{-1}\left(\frac{x}{a}-1\right)$ the value of *n* is

(a)1 (b) – 1 (c) 0 (d) $\frac{1}{2}$

Solution : (c)Let *x* = length $\therefore [X]=[L]$ and $[dx]=[L]$

By principle of dimensional homogeneity $\left[\frac{x}{a}\right]=$ dimensionless $\therefore [a]=[x]=[L]$

By substituting dimension of each quantity in both sides: $\frac{[L]}{[L^2-L^2]^{1/2}}=[L^n]$ $\therefore n=0$

Problem **19.** A physical quantity $P=\frac{B^2l^2}{m}$ where *B*= magnetic induction, *l*= length and *m* = mass. The dimension of *P* is

(a) MLT^{-3} (b) $ML^2T^{-4}I^{-2}$ (c) $M^2L^2T^{-4}I$ (d) $MLT^{-2}I^{-2}$

Solution : (b)*F* = *BIL* $\therefore$ Dimension of $[B]=\frac{[F]}{[I][L]}=\frac{[MLT^{-2}]}{[I][L]}=[MT^{-2}I^{-1}]$

Now dimension of $[P]=\frac{B^2l^2}{m}=\frac{[MT^{-2}I^{-1}]^2\times[L^2]}{[M]}=[ML^2T^{-4}I^{-2}]$

Problem **20.** The equation of the stationary wave is *y*= $2a\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong

(a)The unit of ct is same as that of λ (b) The unit of *x* is same as that of λ

(c)The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$ (d) The unit of c/λ is same as that of x/λ

Solution : (d)Here, $\frac{2\pi ct}{\lambda}$ as well as $\frac{2\pi x}{\lambda}$ are dimensionless (angle) *i.e.* $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

So (i) unit of *c t* is same as that of *λ* (ii) unit of *x* is same as that of *λ* (iii) $\left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$ and (iv) $\frac{x}{\lambda}$ is unit less. It is not the case with $\frac{c}{\lambda}$.

(3) **To convert a physical quantity from one system to the other** : The measure of a physical quantity is *nu* = constant

If a physical quantity *X* has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^aL_1^bT_1^c]$ and $[M_2^aL_2^bT_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$\Rightarrow n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$

$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Example : (1) conversion of *Newton* into *Dyne*.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 *N* = 1 $kg\text{-}m/sec^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 1\left[\frac{10^3 gm}{gm}\right]^1\left[\frac{10^2 cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$

∴ 1 *N* = 10^5 *Dyne*

(2) Conversion of gravitational constant (*G*) from C.G.S. to M.K.S. system

The value of *G* in C.G.S. system is 6.67 × 10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So *G* = 6.67 × 10^{-8} $cm^3/g\,s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times10^{-8}\left[\frac{gm}{10^3 gm}\right]^{-1}\left[\frac{cm}{10^2 cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$$

∴ *G* = 6.67 × 10^{-11} M.K.S. units

Sample problems based on conversion

Problem **21.** Which relation is wrong

(a)1 *Calorie* = 4.18 *Joules* (b) 1Å = 10^{-10} *m*

(c)1 *MeV* = 1.6 × 10^{-13} *Joules* (d) 1 *Newton* = 10^{-5} *Dynes*

Solution : (d)Because 1 *Newton* = 10^5 *Dyne*.

Problem **22.** To determine the Young's modulus of a wire, the formula is $Y = \frac{F}{A}.\frac{L}{\Delta l}$; where *L*= length, A= area of cross- section of the wire, ΔL = Change in length of the wire when stretched with a force *F*. The conversion factor to change it from CGS to MKS system is

(a) 1 (b) 10 (c) 0.1 (d) 0.01

Solution : (c) We know that the dimension of young's modulus is $[ML^{-1}T^{-2}]$

C.G.S. unit : $gm\ cm^{-1}\sec^{-2}$ and M.K.S. unit : $kg.\ m^{-1}\ sec^{-2}$.

By using the conversion formula: $n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-1}\left[\frac{T_1}{T_2}\right]^{-2} = \left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^{-1}\left[\frac{\sec}{\sec}\right]^{-2}$

∴ Conversion factor $\frac{n_2}{n_1} = \left[\frac{gm}{10^3 gm}\right]^1\left[\frac{cm}{10^2 cm}\right]^{-1}\left[\frac{\sec}{\sec}\right]^{-2} = \frac{1}{10} = 0.1$

Problem **23.** A physical quantity is measured and its value is found to be nu where $n =$ numerical value and $u =$ unit.

Then which of the following relations is true

(a) $n \propto u^2$ (b) $n \propto u$ (c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Solution : (d) We know $P = nu =$ constant ∴ $n_1u_1 = n_2u_2$ or $n \propto \frac{1}{u}$.

Problem **24.** In C.G.S. system the magnitude of the force is 100 *dynes*. In another system where the fundamental physical quantities are *kilogram*, *metre* and *minute*, the magnitude of the force is **[EAMCET 2001]**

(a) 0.036 (b) 0.36 (c) 3.6 (d) 36

Solution : (c) $n_1 = 100$, $M_1 = g$, $L_1 = cm$, $T_1 = \sec$ and $M_2 = kg$, $L_2 = meter$, $T_2 = minute$, $x = 1$, $y = 1$, $z = -2$

By substituting these values in the following conversion formula $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$

$$n_2 = 100\left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^1\left[\frac{\sec}{minute}\right]^{-2}$$

$$n_2 = 100\left[\frac{gm}{10^3 gm}\right]^1\left[\frac{cm}{10^2 cm}\right]^1\left[\frac{\sec}{60\sec}\right]^{-2} = 3.6$$

Problem **25.** The temperature of a body on Kelvin scale is found to be *X K*. When it is measured by a Fahrenheit thermometer, it is found to be *X F*. Then *X* is **[UPSEAT 200]**

(a) 301.25 (b) 574.25 (c) 313 (d) 40

Solution : (c) Relation between centigrade and Fahrenheit $\frac{K-273}{5} = \frac{F-32}{9}$

According to problem $\frac{X-273}{5} = \frac{X-32}{9}$ ∴ $X = 313$.

Problem **26.** Conversion of 1 MW power on a new system having basic units of mass, length and time as 10*kg*, 1*dm* and 1 *minute* respectively is

(a) $2.16\times10^{12} unit$ (b) $1.26\times10^{12} unit$ (c) $2.16\times10^{10} unit$ (d) $2\times10^{14} unit$

Solution : (a) $[P] = [ML^2T^{-3}]$

Using the relation $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$ $= 1\times10^6\left[\frac{1kg}{10kg}\right]^1\left[\frac{1m}{1dm}\right]^2\left[\frac{1s}{1\min}\right]^{-3}$

[As $1MW = 10^6 W$]

$= 10^6\left[\frac{1kg}{10kg}\right]\left[\frac{10dm}{1dm}\right]^2\left[\frac{1sec}{60sec}\right]^{-3}$ $= 2.16\times10^{12}$ *unit*

Problem **27.** In two systems of relations among velocity, acceleration and force are respectively $v_2 = \frac{\alpha^2}{\beta}v_1$, $a_2 = \alpha\beta a_1$ and $F_2 = \frac{F_1}{\alpha\beta}$. If α and β are constants then relations among mass, length and time in two systems are

(a) $M_2 = \frac{\alpha}{\beta}M_1, L_2 = \frac{\alpha^2}{\beta^2}L_1, T_2 = \frac{\alpha^3 T_1}{\beta}$ (b) $M_2 = \frac{1}{\alpha^2\beta^2}M_1, L_2 = \frac{\alpha^3}{\beta^3}L_1, T_2 = T_1\frac{\alpha}{\beta^2}$

(c) $M_2 = \frac{\alpha^3}{\beta^3}M_1, L_2 = \frac{\alpha^2}{\beta^2}L_1, T_2 = \frac{\alpha}{\beta}T_1$ (d) $M_2 = \frac{\alpha^2}{\beta^2}M_1, L_2 = \frac{\alpha}{\beta^2}L_1, T_2 = \frac{\alpha^3}{\beta^3}T_1$

Solution : (b) $v_2 = v_1\frac{\alpha^2}{\beta}$ $\Rightarrow [L_2T_2^{-1}] = [L_1T_1^{-1}]\frac{\alpha^2}{\beta}$ ......(i)

$a_2 = a_1\alpha\beta$ $\Rightarrow [L_2T_2^{-2}] = [L_1T_1^{-2}]\alpha\beta$ ......(ii)

and $F_2 = \frac{F_1}{\alpha\beta}$ $\Rightarrow [M_2L_2T_2^{-2}] = [M_1L_1T_1^{-2}]\times\frac{1}{\alpha\beta}$ ......(iii)

Dividing equation (iii) by equation (ii) we get $M_2 = \frac{M_1}{(\alpha\beta)\alpha\beta} = \frac{M_1}{\alpha^2B^2}$

Squaring equation (i) and dividing by equation (ii) we get $L_2 = L_1\frac{\alpha^3}{\beta^3}$

Dividing equation (i) by equation (ii) we get $T_2 = T_1\frac{\alpha}{\beta^2}$

Problem **28.** If the present units of length, time and mass (*m*, *s*, *kg*) are changed to 100*m*, 100*s*, and $\frac{1}{10}$ *kg* then

(a)The new unit of velocity is increased 10 times

(b)The new unit of force is decreased $\frac{1}{1000}$ times

(c)The new unit of energy is increased 10 times

(d)The new unit of pressure is increased 1000 times

Solution : (b) Unit of velocity = *m/sec* ; in new system = $\frac{100m}{100\sec} = \frac{m}{\sec}$ (same)

Unit of force $= \frac{kg\times m}{\sec^2}$; in new system $= \frac{1}{10}kg\times\frac{100m}{100\sec\times100\sec} = \frac{1}{1000}\frac{kg\times m}{\sec^2}$

Unit of energy $= \frac{kg\times m^2}{\sec^2}$; in new system $= \frac{1}{10}kg\times\frac{100m\times100m}{100\sec\times100\sec} = \frac{1}{10}\frac{kg\times m^2}{sec^2}$

Unit of pressure $= \frac{kg}{m\times\sec^2}$; in new system $= \frac{1}{10}kg\times\frac{1}{100}m\times\frac{1}{100sec\times100sec} = 10^{-7}\frac{kg}{m\times sec^2}$

Problem **29.** Suppose we employ a system in which the unit of mass equals 100 *kg*, the unit of length equals 1 *km* and the unit of time 100 *s* and call the unit of energy *eluoj* (*joule* written in reverse order), then

(a) 1 *eluoj* = 10^4 *joule* (b) 1 *eluoj* = 10^{-3} *joule* (c) 1 *eluoj* = 10^{-4} *joule* (d) 1 *joule* = 10^3 *eluoj*

Solution : (a) $[E]=[ML^2T^{-2}]$

1 *eluoj* $=[100kg]\times[1km]^2\times[100\sec]^2 = 100kg\times10^6m^2\times10^{-4}\sec^{-2} = 10^4\,kgm^2\times\sec^{-2} = 10^4\,Joule$

Problem **30.** If $1gm\,cms^{-1} = x\,Ns$, then number *x* is equivalent to

(a) 1×10^{-1} (b) 3×10^{-2} (c) 6×10^{-4} (d) 1×10^{-5}

Solution : (d) $gm\text{-}cms^{-1} = 10^{-3}kg\times10^{-2}m\times s^{-1} = 10^{-5}kg\times m\times s^{-1} = 10^{-5}\,Ns$

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(4) **To check the dimensional correctness of a given physical relation :** This is based on the '*principle of homogeneity*'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

then according to principle of homogeneity $[X] = [A] = [(BC)^2] = [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Example : (1) $F = mv^2/r^2$

By substituting dimension of the physical quantities in the above relation –

$[MLT^{-2}] = [M][LT^{-1}]^2/[L]^2$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation –

$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$

i.e. $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

Sample problems based on formulae checking

Problem **31.** From the ~~dimensional consideration, which of the follow~~ing equation is correct **[CPMT 1983]**

(a) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\frac{GM}{R^3}}$ (c) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

Solution : (a) $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $GM = gR^2$]

Now by substituting the dimension of each quantity in both sides.

$[T] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T]$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct**.**

Problem **32.** A highly rigid cubical block *A* of small mass *M* and side *L* is fixed rigidly onto another cubical block *B* of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of *B*. The lower face of B is rigidly held on a horizontal surface. A small force *F* is applied perpendicular to one of the side faces of *A*. After the force is withdrawn block *A* executes small oscillations. The time period of which is given by

[IIT-JEE 1992]

(a) $2\pi\sqrt{\frac{M\eta}{L}}$ (b) $2\pi\sqrt{\frac{L}{M\eta}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Solution : (d) Given *m* = mass = [*M*], η = coefficient of rigidity = $[ML^{-1}T^{-2}]$, L = length = [*L*]

By substituting the dimension of these quantity we can check the accuracy of the given formulae

$$[T]=2\pi\left(\frac{[M]}{[\eta][L]}\right)^{1/2} = \left[\frac{M}{ML^{-1}T^{-2}L}\right]^{1/2} = [T].$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem **33.** A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity. After some time the velocity of the ball attains a constant value known as terminal velocity v_T. The terminal velocity depends on (i) the mass of the ball. (ii) η (iii) *r* and (iv) acceleration due to gravity *g*. which of the following relations is dimensionally correct

(a) $v_T \propto \frac{mg}{\eta r}$ (b) $v_T \propto \frac{\eta r}{mg}$ (c) $v_T \propto \eta rmg$ (d) $v_T \propto \frac{mgr}{\eta}$

Solution : (a) Given v_T = terminal velocity = $[LT^{-1}]$, *m* = Mass = [*M*], *g* = Acceleration due to gravity = $[LT^{-2}]$

r = Radius = [*L*], η = Coefficient of viscosity = $[\eta]$

By substituting the dimension of each quantity we can check the accuracy of given formula $v_T \propto \frac{mg}{\eta r}$

$$\therefore [LT^{-1}]=\frac{[M][LT^{-2}]}{[ML^{-1}T^{-1}][L]} = [LT^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem **34.** A dimensionally consistent relation for the volume *V* of a liquid of coefficient of viscosity η flowing per second through a tube of radius r and length *l* and having a pressure difference p across its end, is

(a) $V=\frac{\pi pr^4}{8\eta l}$ (b) $V=\frac{\pi\eta l}{8pr^4}$ (c) $V=\frac{8p\eta l}{\pi r^4}$ (d) $V=\frac{\pi p\eta}{8lr^4}$

Solution : (a) Given *V* = Rate of flow = $\frac{\text{Volume}}{\text{sec}}=[L^3T^{-1}]$, *P* = Pressure = $[ML^{-1}T^{-2}]$, *r* = Radius = [*L*]

η = Coefficient of viscosity = $[ML^{-1}T^{-1}]$, *l* = Length = [*L*]

By substituting the dimension of each quantity we can check the accuracy of the formula $V=\frac{\pi P r^4}{8\eta l}$

$$\therefore [L^3T^{-1}]=\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} = [L^3T^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem **35.** With the usual notations, the following equation $S_t = u+\frac{1}{2}a(2t-1)$ is

(a) Only numerically correct (b) Only dimensionally correct
(c) Both numerically and dimensionally correct (d) Neither numerically nor dimensionally correct

Solution : (c) Given S_t = distance travelled by the body in t^{th} *sec*.= $[LT^{-1}]$, a = Acceleration = $[LT^{-2}]$, v = velocity = $[LT^{-1}]$, t = time = $[T]$
By substituting the dimension of each quantity we can check the accuracy of the formula

$$S_t = u + \frac{1}{2}a(2t-1)$$

$$\therefore [LT^{-1}] = [LT^{-1}] + [LT^{-2}][T] \Rightarrow [LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

Since the dimension of each terms are equal therefore this equation is dimensionally correct. And after deriving this equation from Kinematics we can also proof that this equation is correct numerically also.

Problem **36.** If velocity V, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of V, A and F would be
(a) $FA^{-1}v$ (b) Fv^3A^{-2} (c) Fv^2A^{-1} (d) $F^2v^2A^{-1}$

Solution : (b) Given, v = velocity = $[LT^{-1}]$, A = Acceleration = $[LT^{-2}]$, F = force = $[MLT^{-2}]$
By substituting, the dimension of each quantity we can check the accuracy of the formula
[Angular momentum] = Fv^3A^{-2}

$$[ML^2T^{-1}] = [MLT^{-2}][LT^{-1}]^3[LT^{-2}]^{-2}$$

$$= [ML^2T^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem **37.** The largest mass (m) that can be moved by a flowing river depends on velocity (v), density (ρ) of river water and acceleration due to gravity (g). The correct relation is
(a) $m \propto \frac{\rho^2 v^4}{g^2}$ (b) $m \propto \frac{\rho v^6}{g^2}$ (c) $m \propto \frac{\rho v^4}{g^3}$ (d) $m \propto \frac{\rho v^6}{g^3}$

Solution : (d) Given, m = mass = $[M]$, v = velocity = $[LT^{-1}]$, ρ = density = $[ML^{-3}]$, g = acceleration due to gravity = $[LT^{-2}]$
By substituting, the dimension of each quantity we can check the accuracy of the formula

$$m = K\frac{\rho v^6}{g^3}$$

$$\Rightarrow [M] = \frac{[ML^{-3}][LT^{-1}]^6}{[LT^{-2}]^3}$$

$$= [M]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

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(5) **As a research tool to derive new relations :** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.
Example : (i) Time period of a simple pendulum.
Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then assuming the function to be product of power function of m, l and g
i.e., $T = Km^xl^yg^z$; where K = dimensionless constant
If the above relation is dimensionally correct then by substituting the dimensions of quantities –

$[T] = [M]^x [L]^y [LT^{-2}]^z$

or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$

Equating the exponents of similar quantities $x = 0$, $y = 1/2$ and $z = -1/2$

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2π) through experiments so $T = 2\pi\sqrt{\frac{l}{g}}$

(ii) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v and the viscosity η of the fluid.

So $F = f(\eta, r, v)$

If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where K is dimensionless constant.

If the above relation is dimensionally correct $[MLT^{-2}] = [ML^{-1}T^{-1}]^x [L]^y [LT^{-1}]^z$

or $[MLT^{-2}] = [M^xL^{-x+y+z}T^{-x-z}]$

Equating the exponents of similar quantities $x = 1$; $-x + y + z = 1$ and $-x - z = -2$

Solving these for x, y and z, we get $x = y = z = 1$

So eq^n (i) becomes $F = K\eta rv$

On experimental grounds, $K = 6\pi$; so $F = 6\pi\eta rv$

This is the famous Stoke's law.

Sample problem based on formulae derivation

Problem **38.** If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is

[UPSEAT 2002]

(a) $c^{1/2}G^{1/2}h^{1/2}$ (b) $c^{1/2}G^{1/2}h^{-1/2}$ (c) $c^{1/2}G^{-1/2}h^{1/2}$ (d) $c^{-1/2}G^{1/2}h^{1/2}$

Solution : (c) Let $m \propto c^xG^yh^z$ or $m = K\,c^xG^yh^z$

By substituting the dimension of each quantity in both sides

$[M^1L^0T^0] = K[LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^2T^{-1}]^z = [M^{-y+z}L^{x+3y+2z}T^{-x-2y-z}]$

By equating the power of M, L and T in both sides : $-y + z = 1$, $x + 3y + 2z = 0$, $-x - 2y - z = 0$

By solving above three equations $x = 1/2$, $y = -1/2$ and $z = 1/2$.

$\therefore\ m \propto c^{1/2}G^{-1/2}h^{1/2}$

Problem **39.** If the time period (T) of vibration of a liquid drop depends on surface tension (S), radius (r) of the drop and density (ρ) of the liquid, then the expression of T is

[AMU (Med.) 2000]

(a) $T = K\sqrt{\rho r^3/S}$ (b) $T = K\sqrt{\rho^{1/2}r^3/S}$ (c) $T = K\sqrt{\rho r^3/S^{1/2}}$ (d) None of these

Solution : (a) Let $T \propto S^xr^y\rho^z$ or $T = K\,S^xr^y\rho^z$

By substituting the dimension of each quantity in both sides

$[M^0L^0T^1] = K\,[MT^{-2}]^x[L]^y[ML^{-3}]^z = [M^{x+z}L^{y-3z}T^{-2x}]$

By equating the power of M, L and T in both sides $x + z = 0$, $y - 3z = 0$, $-2x = 1$

By solving above three equations $\therefore\ x = -1/2$, $y = 3/2$, $z = 1/2$

So the time period can be given as, $T = K\,S^{-1/2}r^{3/2}\rho^{1/2} = K\sqrt{\frac{\rho r^3}{S}}$.

***Problem* 40.** If P represents radiation pressure, C represents speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers *x, y* and *z* such that $P^xQ^yC^z$ is dimensionless, are

(a) $x=1, y=1, z=-1$ (b) $x=1, y=-1, z=1$ (c) $x=-1, y=1, z=1$ (d) $x=1, y=1, z=1$

Solution : (b) $[P^xQ^yC^z]=M^0L^0T^0$

By substituting the dimension of each quantity in the given expression

$[ML^{-1}T^{-2}]^x[MT^{-3}]^y[LT^{-1}]^z = [M^{x+y}L^{-x+z}T^{-2x-3y-z}]=M^0L^0T^0$

by equating the power of *M, L* and *T* in both sides: $x+y=0$, $-x+z=0$ and $-2x-3y-z=0$

by solving we get $x=1, y=-1, z=1$.

***Problem* 41.** The volume V of water passing through a point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation $V \propto A^\alpha u^\beta t^\gamma$, which one of the following will be true

(a) $\alpha=\beta=\gamma$ (b) $\alpha\neq\beta=\gamma$ (c) $\alpha=\beta\neq\gamma$ (d) $\alpha\neq\beta\neq\gamma$

Solution : (b) Writing dimensions of both sides $[L^3]=[L^2]^\alpha[LT^{-1}]^\beta[T]^\gamma \Rightarrow [L^3T^0]=[L^{2\alpha+\beta}T^{\gamma-\beta}]$

By comparing powers of both sides $2\alpha+\beta=3$ and $\gamma-\beta=0$

Which give $\beta=\gamma$ and $\alpha=\frac{1}{2}(3-\beta)$ *i.e.* $\alpha\neq\beta=\gamma$.

***Problem* 42.** If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

(a) $V^{-2}F^0E$ (b) V^0FE^2 (c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

Solution : (d) Let $M=V^aF^bE^c$

Putting dimensions of each quantities in both side $[M]=[LT^{-1}]^a[MLT^{-2}]^b[ML^2T^{-2}]^c$

Equating powers of dimensions. We have $b+c=1$, $a+b+2c=0$ and $-a-2b-2c=0$

Solving these equations, $a=-2$, $b=0$ and $c=1$

So $M=[V^{-2}F^0E]$

***Problem* 43.** Given that the amplitude A of scattered light is :

(i) Directly proportional to the amplitude (A_0) of incident light.

(ii) Directly proportional to the volume (V) of the scattering particle

(iii) Inversely proportional to the distance (r) from the scattered particle

(iv) Depend upon the wavelength (λ) of the scattered light. then:

(a) $A\propto\frac{1}{\lambda}$ (b) $A\propto\frac{1}{\lambda^2}$ (c) $A\propto\frac{1}{\lambda^3}$ (d) $A\propto\frac{1}{\lambda^4}$

Solution : (b) Let $A=\frac{KA_0V\lambda^x}{r}$

By substituting the dimension of each quantity in both sides

$\Rightarrow[L]=\frac{[L].[L^3][L^x]}{[L]}$

$\therefore [L]=[L^{3+x}]$; $\Rightarrow 3+x=1$ or $x=-2$

$\therefore\ A \propto \lambda^{-2}$

1.12 Limitations of Dimensional Analysis

Although dimensional analysis is very useful it cannot lead us too far as,

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.

(2) Numerical constant having no dimensions [*K*] such as (1/2), 1 or 2π *etc*. cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \quad \text{or} \quad y = a\sin\omega t$$

cannot be derived by using this theory (try if you can). However, the dimensional correctness of these can be checked.

(4) The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than 3 physical quantities as then there will be less number (= 3) of equations than the unknowns (>3). However still we can check correctness of the given equation dimensionally. For example $T = 2\pi\sqrt{1/mgl}$ can not be derived by theory of dimensions but its dimensional correctness can be checked.

(5) Even if a physical quantity depends on 3 physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions, *e.g.*, formula for the frequency of a tuning fork $f = (d/L^2)v$ cannot be derived by theory of dimensions but can be checked.

1.13 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.

Example : 42.3 has three significant figures.
243.4 has four significant figures.
24.123 has five significant figures.

(2) A zero becomes significant figure if it appears between to non-zero digits.

Example : 5.03 has three significant figures.
5.604 has four significant figures.
4.004 has four significant figures.

(3) Leading zeros or the zeros placed to the left of the number are never significant.

Example : 0.543 has three significant figures.
0.045 has two significant figures.
0.006 has one significant figures.

(4) Trailing zeros or the zeros placed to the right of the number are significant.

Example : 4.330 has four significant figures.
433.00 has five significant figures.
343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.

Example : 1.32×10^{-2} has three significant figures.
1.32×10^4 has three significant figures.

1.14 Rounding Off

While rounding off measurements, we use the following rules by convention:
(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
Example : $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.
(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.
Example : x = 6.87 is rounded off to 6.9, again x = *12.78* is rounded off to 12.8.
(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.
Example : x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.
(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.
Example : x = 3.250 becomes 3.2 on rounding off, again x = 12.650 becomes 12.6 on rounding off.
(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.
Example : x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

1.15 Significant Figures in Calculation

In most of the experiments, the observations of various measurements are to be combined mathematically, *i.e.*, added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.
The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples :

(i)
33.3 ← (has only one decimal place)
3.11
+ 0.313
36.723 ← (answer should be reported to one decimal place)
Answer = 36.7

(ii)
3.1421
0.241
+ 0.09 ← (has 2 decimal places)
3.4731 ← (answer should be reported to 2 decimal places)
Answer = 3.47

(iii)
62.831 ← (has 3 decimal places)
– 24.5492
38.2818 ← (answer should be reported to 3 decimal places after rounding off)
Answer = 38.282

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples :

(i)
142.06
× 0.23 ← (two significant figures)
32.6738 ← (answer should have two significant figures)
Answer = 33

(ii)
51.028
× 1.31 ← (three significant figures)
66.84668

Answer = 66.8

(iii) $\frac{0.90}{4.26} = 0.211267$

Answer = 0.21

1.16 Order of Magnitude

In scientific notation the numbers are expressed as, Number $= M \times 10^x$. Where M is a number lies between 1 and 10 and *x* is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(1) Speed of light in vacuum $= 3 \times 10^8 ms^{-1} \approx 10^8 m/s$ (ignoring 3 < 5)

(2) Mass of electron $= 9.1 \times 10^{-31} kg \approx 10^{-30} kg$ (as 9.1 > 5).

Sample problems based on significant figures

***Problem* 44.** Each side ~~a cube is measured to be 7.203 *m*. The volume~~ of the cube up to appropriate significant figures is

(a) 373.714 (b) 373.71 (c) 373.7 (d) 373

Solution : (c) Volume $= a^3 = (7.023)^3 = 373715 m^3$

In significant figures volume of cube will be $3737 m^3$ because its side has four significant figures.

***Problem* 45.** The number of significant figures in 0.007 m^2 is

(a) 1 (b) 2 (c) 3 (d) 4

Solution : (a)

***Problem* 46.** The length, breadth and thickness of a block are measured as 125.5 *cm*, 5.0 *cm* and 0.32 *cm* respectively. Which one of the following measurements is most accurate

(a) Length (b) Breadth (c) Thickness (d) Height

Solution : (a) Relative error in measurement of length is minimum, so this measurement is most accurate.

***Problem* 47.** The mass of a box is 2.3 kg. Two marbles of masses 2.15 *g* and 12.39 *g* are added to it. The total mass of the box to the correct number of significant figures is

(a) 2.340 *kg* (b) 2.3145 *kg*. (c) 2.3 *kg* (d) 2.31 *kg*

Solution : (c) Total mass $= 2.3 + 0.00215 + 0.01239 = 2.31 kg$

Total mass in appropriate significant figures be 2.3 *kg*.

***Problem* 48.** The length of a rectangular sheet is 1.5 cm and breadth is 1.203 cm. The area of the face of rectangular sheet to the correct no. of significant figures is :

(a) 1.8045 cm^2 (b) 1.804 cm^2 (c) 1.805 cm^2 (d) 1.8 cm^2

Solution : (d) Area $= 1.5 \times 1.203 = 1.8045 cm^2 = 1.8 cm^2$ (Upto correct number of significant figure).

***Problem* 49.** Each side of a cube is measured to be 5.402 cm. The total surface area and the volume of the cube in appropriate significant figures are :

(a) 175.1 cm^2, 157 cm^2 (b) 175.1 cm^2, 157.6 cm^3

(c) 175 cm^2, 157 cm^2 (d) 175.08 cm^2, 157.639 cm^3

Solution : (b) Total surface area = $6 \times (5.402)^2 = 175.09 cm^2 = 175.1 cm^2$ (Upto correct number of significant figure)

Total volume $= (5.402)^3 = 157.64 cm^3 = 175.6 cm^3$ (Upto correct number of significant figure).

***Problem* 50.** Taking into account the significant figures, what is the value of 9.99 *m* + 0.0099 *m*

(a) 10.00 *m* (b) 10 *m* (c) 9.9999 *m* (d) 10.0 *m*

Solution : (a) $9.99m + 0.0099m = 9.999m \quad = 10.00m$ (In proper significant figures).

***Problem* 51.** The value of the multiplication 3.124 × 4.576 correct to three significant figures is

(a) 14.295 (b) 14.3 (c) 14.295424 (d) 14.305

Solution : (b) $3.124 \times 4.576 = 14.295$ =14.3 (Correct to three significant figures).

***Problem* 52.** The number of the significant figures in 11.118×10^{-6} *V* is

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (c) The number of significant figure is 5 as 10^{-6} does not affect this number.

***Problem* 53.** If the value of resistance is 10.845 *ohms* and the value of current is 3.23 *amperes*, the potential difference is 35.02935 *volts*. Its value in significant number would be

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(a) 35 *V* (b) 35.0 *V* (c) 35.03 *V* (d) 35.025 *V*

Solution : (b) Value of current (3.23 *A*) has minimum significant figure (3) so the value of potential difference $V(=IR)$ have only 3 significant figure. Hence its value be 35.0 *V*.

1.17 Errors of Measurement

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) **Absolute error :** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured *n* times. Let the measured value be $a_1, a_2, a_3, \ldots.. a_n$. The arithmetic mean of these value is $a_m = \dfrac{a_1 + a_2 + \ldots a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

$$\ldots\ldots\ldots\ldots$$

$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) **Mean absolute error :** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots..|\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

(3) **Relative error or Fractional error :** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured. Thus

Relative error or Fractional error $= \dfrac{\text{mean absolute error}}{\text{mean value}} = \dfrac{\overline{\Delta a}}{a_m}$

(4) **Percentage error :** When the relative/fractional error is expressed in percentage, we call it percentage error. Thus

Percentage error $= \frac{\overline{\Delta a}}{a_m} \times 100\%$

1.18 Propagation of Errors

(1) **Error in sum of the quantities :** Suppose $x = a + b$
Let Δa = absolute error in measurement of a
Δb = absolute error in measurement of b
Δx = absolute error in calculation of x *i.e.* sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a + b} \times 100\%$

(2) **Error in difference of the quantities :** Suppose $x = a - b$
Let Δa = absolute error in measurement of a,
Δb = absolute error in measurement of b
Δx = absolute error in calculation of x *i.e.* difference of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a - b} \times 100\%$

(3) **Error in product of quantities :** Suppose $x = a \times b$
Let Δa = absolute error in measurement of a,
Δb = absolute error in measurement of b
Δx = absolute error in calculation of x *i.e.* product of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(4) **Error in division of quantities :** Suppose $x = \frac{a}{b}$
Let Δa = absolute error in measurement of a,
Δb = absolute error in measurement of b
Δx = absolute error in calculation of x *i.e.* division of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(5) **Error in quantity raised to some power :** Suppose $x = \frac{a^n}{b^m}$
Let Δa = absolute error in measurement of a,
Δb = absolute error in measurement of b
Δx = absolute error in calculation of x

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

Percentage error in the value of x = n (Percentage error in value of a) + m (Percentage error in value of b)

Note : ❑ The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.

Sample problems based on errors of measurement

***Problem* 54.** A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^{\alpha}c^{\beta}/d^{\gamma}e^{\delta}$. If the maximum errors in the measurement of b, c, d and e are b_1 %, c_1 %, d_1 % and e_1 %, then the maximum error in the value of a determined by the experiment is

(a) $(b_1 + c_1 + d_1 + e_1)$% (b) $(b_1 + c_1 - d_1 - e_1)$%

(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)$% (d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)$%

Solution : (d) $a = b^{\alpha} c^{\beta} / d^{\gamma} e^{\delta}$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a}\times 100\right)_{max} = \alpha.\frac{\Delta b}{b}\times 100 + \beta.\frac{\Delta c}{c}\times 100 + \gamma.\frac{\Delta d}{d}\times 100 + \delta.\frac{\Delta e}{e}\times 100$$

$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

***Problem* 55.** The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, The maximum error in the measurement of pressure is
]

(a) 1% (b) 2% (c) 6% (d) 8%

Solution : (d) $P = \frac{F}{A} = \frac{F}{l^2}$, so maximum error in pressure (P)

$$\left(\frac{\Delta P}{P}\times 100\right)_{max} = \frac{\Delta F}{F}\times 100 + 2\frac{\Delta l}{l}\times 100$$ = 4% + 2 × 2% = 8%

***Problem* 56.** The relative density of material of a body is found by weighing it first in air and then in water. If the weight in air is (5.00 ± 0.05) *Newton* and weight in water is (4.00 ± 0.05) *Newton*. Then the relative density along with the maximum permissible percentage error is

(a) 5.0 ± 11% (b) 5.0 ± 1% (c) 5.0 ± 6% (d) 1.25 ± 5%

Solution : (a) Weight in air $= (5.00 \pm 0.05)N$

Weight in water $= (4.00 \pm 0.05)N$

Loss of weight in water $= (1.00 \pm 0.1)N$

Now relative density $= \frac{\text{weight in air}}{\text{weight loss in water}}$ *i.e. R . D* $= \frac{5.00 \pm 0.05}{1.00 \pm 0.1}$

Now relative density with max permissible error $= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00}\right)\times 100$ $= 5.0 \pm (1 + 10)\%$
$= 5.0 \pm 11\%$

***Problem* 57** The resistance $R = \frac{V}{i}$ where V = 100 ± 5 volts and i = 10 ± 0.2 *amperes*. What is the total error in R

(a) 5% (b) 7% (c) 5.2% (d) $\frac{5}{2}$ %

Solution : (b) $R = \frac{V}{I}$ $\therefore \left(\frac{\Delta R}{R}\times 100\right)_{max} = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100 = \frac{5}{100}\times 100 + \frac{0.2}{10}\times 100 = (5+2)\%$ = 7%

***Problem* 58.** The period of oscillation of a simple pendulum in the experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. The average absolute error is

(a) 0.1 s (b) 0.11 s (c) 0.01 s (d) 1.0 s

Solution : (b) Average value $= \dfrac{2.63+2.56+2.42+2.71+2.80}{5} = 2.62 sec$

Now $|\Delta T_1| = 2.63 - 2.62 = 0.01$

$|\Delta T_2| = 2.62 - 2.56 = 0.06$

$|\Delta T_3| = 2.62 - 2.42 = 0.20$

$|\Delta T_4| = 2.71 - 2.62 = 0.09$

$|\Delta T_5| = 2.80 - 2.62 = 0.18$

Mean absolute error $\Delta T = \dfrac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5} = \dfrac{0.54}{5} = 0.108 = 0.11 sec$

***Problem* 59.** The length of a cylinder is measured with a meter rod having least count 0.1 *cm*. Its diameter is measured with venier calipers having least count 0.01 *cm*. Given that length is 5.0 *cm*. and radius is 2.0 *cm*. The percentage error in the calculated value of the volume will be

(a) 1% (b) 2% (c) 3% (d) 4%

Solution : (c) Volume of cylinder $V = \pi r^2 l$

Percentage error in volume $\dfrac{\Delta V}{V} \times 100 = \dfrac{2\Delta r}{r} \times 100 + \dfrac{\Delta l}{l} \times 100$

$= \left(2 \times \dfrac{0.01}{2.0} \times 100 + \dfrac{0.1}{5.0} \times 100\right) = (1+2)\% = 3\%$

***Problem* 60.** In an experiment, the following observation's were recorded : $L = 2.820$ *m*, $M = 3.00$ *kg*, $l = 0.087$ *cm*, Diameter $D = 0.041$ *cm* Taking $g = 9.81$ m/s^2 using the formula , $Y = \dfrac{4Mg}{\pi D^2 l}$, the maximum permissible error in Y is

(a) 7.96% (b) 4.56% (c) 6.50% (d) 8.42%

Solution : (c) $Y = \dfrac{4MgL}{\pi D^2 l}$ so maximum permissible error in Y =

$\dfrac{\Delta Y}{Y} \times 100 = \left(\dfrac{\Delta M}{M} + \dfrac{\Delta g}{g} + \dfrac{\Delta L}{L} + \dfrac{2\Delta D}{D} + \dfrac{\Delta l}{l}\right) \times 100$

$= \left(\dfrac{1}{300} + \dfrac{1}{9.81} + \dfrac{1}{9820} + 2 \times \dfrac{1}{41} + \dfrac{1}{87}\right) \times 100 = 0.065 \times 100 = 6.5\%$

***Problem* 61.** According to Joule's law of heating, heat produced $H = I^2 Rt$, where I is current, R is resistance and t is time. If the errors in the measurement of I, R and t are 3%, 4% and 6% respectively then error in the measurement of H is

(a) ± 17% (b) ± 16% (c) ± 19% (d) ± 25%

Solution : (b) $H = I^2 Rt$

$\therefore \dfrac{\Delta H}{H} \times 100 = \left(\dfrac{2\Delta I}{I} + \dfrac{\Delta R}{R} + \dfrac{\Delta t}{t}\right) \times 100 = (2 \times 3 + 4 + 6)\% = 16\%$

***Problem* 62.** If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is

(a) 25% (b) 50% (c) 100% (d) 125%

Solution : (c) Kinetic energy $E = \dfrac{1}{2} mv^2$

$\therefore \dfrac{\Delta E}{E} \times 100 = \left(\dfrac{\Delta m}{m} + \dfrac{2\Delta v}{v}\right) \times 100$

Here $\Delta m = 0$ and $\frac{\Delta v}{v} \times 100 = 50\%$

$\therefore \frac{\Delta E}{E} \times 100 = 2 \times 50 = 100\%$

Problem **63.** A physical quantity *P* is given by *P*= $\frac{A^3 B^{\frac{1}{2}}}{C^{-4} D^{\frac{3}{2}}}$. The quantity which brings in the maximum percentage error in *P* is

(a) *A* (b)*B* (c) *C* (d) *D*

Solution : (c) Quantity *C* has maximum power. So it brings maximum error in *P*.