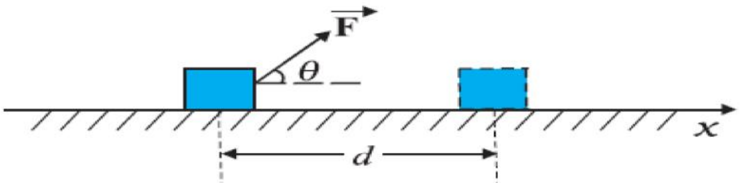
Work, Energy, Power and Collision

6.1 Introduction

WORK

Work is related to force and the displacement over which it acts. Consider a constant force **F** acting on an object of mass *m*. The object undergoes a displacement **d** in the positive *x*-direction as shown

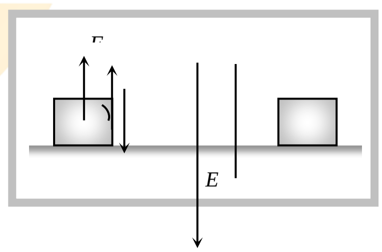


The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

***W* = (*F* cos θ □) *d* = F.d**

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance *s*



By resolving force $\vec{F}$ into two components:

(i) $F\cos\theta$ in the direction of displacement of the body.

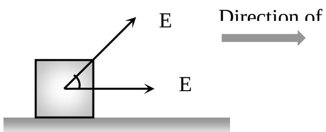
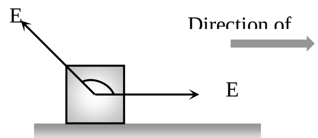
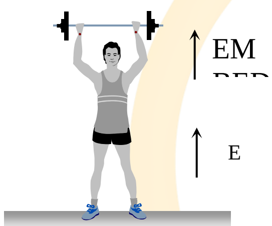
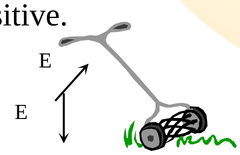
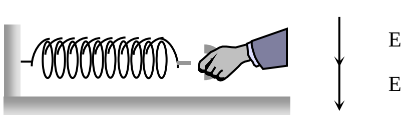
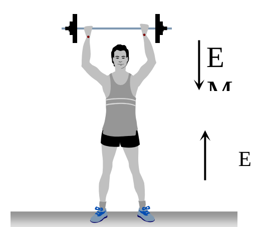
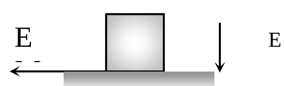
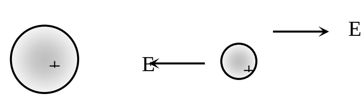
(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body. Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance *s* is given by

$$W = (F\cos\theta)s = Fs\cos\theta \qquad \text{or} \qquad W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar or dot product of the force and the displacement of the body.

If a number of force $\vec{F}_1, \vec{F}_2, \vec{F}_3 ...... \vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then $W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... \vec{F}_n).(\vec{r}_2 - \vec{r}_1)$

6.3 Nature of Work Done

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement  $0^o \le \theta < 90^o$ The positive work signifies that the external force favours the motion of the body.	Negative work means that force (or its component) is opposite to displacement *i.e.*  $90^o < \theta \le 180^o$ The negative work signifies that the external force opposes the motion of the body.
Example: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive  (ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.  (iii) When a spring is stretched, work done by the external (stretching) force is positive.  	*Example*: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative.  (ii) When a body is made to slide over a rough surface, the work done by the frictional force is negative.  (iii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative. 
Maximum work : $W_{max} = F\,s$ When $\cos\theta = \text{maximum} = 1$ *i.e.* $\theta = 0^o$	Minimum work : $W_{min} = -F\,s$ When $\cos\theta = \text{minimum} = -1$ *i.e* $\theta = 180^o$

It means force does maximum work when angle between force and displacement is zero.	It means force does minimum [maximum negative] work when angle between force and displacement is 180^o.

Zero work

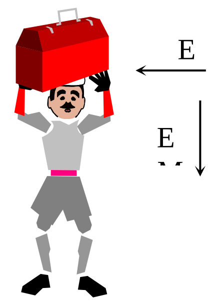
Under three condition, work done becomes zero $W = Fs\cos\theta = 0$

(1) **If the force is perpendicular to the displacement** $[\vec{F} \perp \vec{s}]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done against gravity by the coolie is zero.

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

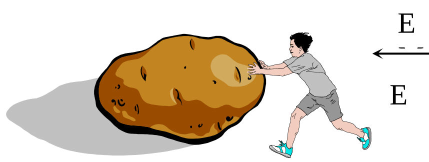
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) **If there is no displacement [*s* = 0]**

Example: (i) When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



4. WORK DONE BY GRAVITY

(i) The work done by the force of gravity on a particle depends only on the initial and final vertical coordinates (because gravity is a vertical force).

(ii) It does not depend on the path taken by the particles.

(iii) The work done by gravity is zero for any path that returns to its initial point.

(iv) The work done by gravity is positive when the body moves downward and it is negative when the body moves upward

A block going up a smooth inclined plane

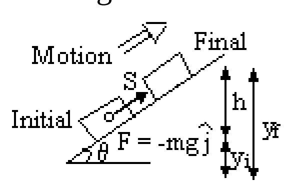
$F = -mg\hat{j}$

$s = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$

W = F.s = -mg$(y_f - y_i)$

Since $s = y_i - y_f$ = h, therefore

W = -mgh



A block slides down a smooth inclined plane

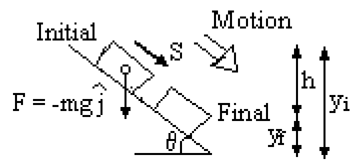
$F = -mg\hat{j}$

$s = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$

W = F.s = -mg$(y_f - y_i)$

Since $s = y_i - y_f$ = h, therefore

W = mgh



Problems based on work done by constant force

***Problem* 1.** A force $F = (5\hat{i} + 3\hat{j})\,N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ *metres*. The work done on the particle is

(a) $-7\,J$ (b) $+13\,J$ (c) $+7\,J$ (d)$+11\,J$

Solution : (c) Work done $= \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7\,J$

***Problem* 2.** A box of mass 1 *kg* is pulled on a horizontal plane of length 1 *m* by a force of 8 *N* then it is raised vertically to a height of 2*m*, the net work done is

(a) 28 *J* (b) 8 *J* (c) 18 *J* (d) None of above

Solution : (a) Work done to displace it horizontally = $F \cdot s = 8 \cdot 1 = 8\,J$

Work done to raise it vertically $F \cdot s = mgh = 1 \cdot 10 \cdot 2 = 20\,J$

4 Net work done = 8 +20 = 28 *J*

NCERT 6.5(d) In Fig. 6.13(i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6.13(ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?

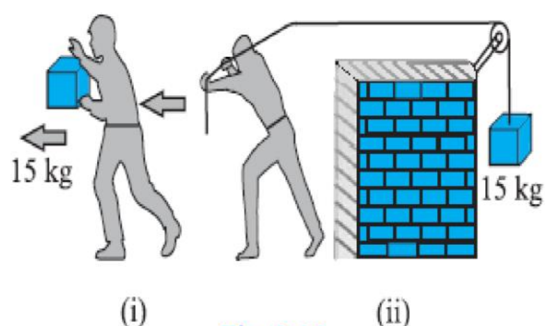


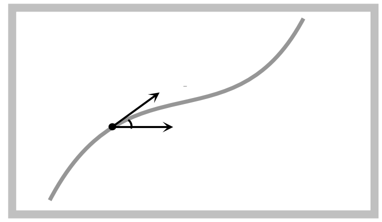
Fig. 6.13

6.4 Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.d\vec{s}$

The total work done in going from A to B as shown in the figure is

$$W = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$$



In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$

Sample Problems based on work done by variable force

***<u>Problem</u>* 3.** A position dependent force $\vec{F} = (7 - 2x + 3x^2)N$ acts on a small abject of mass 2 kg to displace it from $x = 0$ to $x = 5m$. The work done in joule is **[CBSE PMT 1994]**

(a) 70 J (b) 270 J (c) 35 J (d) 135 J

Sol :(d).Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)dx = [7x - x^2 + x^3]_0^5 = 35 - 25 + 125 = 135\ J$

***<u>Problem</u>* 4.** A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

(a) Cx_1^2 (b) $\frac{1}{2}Cx_1^2$ (c) Cx_1 (d) Zero

Solution : (b) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}Cx_1^2$

***<u>Problem</u>* 5.** Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by

(a) $\frac{1}{2}m\frac{v}{t_1}t^2$ (b) $m\frac{v}{t_1}t^2$ (c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

Solution : (d) Work done = $F.s$ = $ma.\left(\frac{1}{2}at^2\right) = \frac{1}{2}ma^2t^2 = \frac{1}{2}m\left(\frac{v}{t_1}\right)^2 t^2$

$$\left[\text{As acceleration } (a) = \frac{v}{t_1} \text{ given}\right]$$

6.5 Dimension and Units of Work

Dimension : As work = Force . displacement

4 $[W]$ = [Force] . [Displacement] $= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

Units : The units of work are of two types

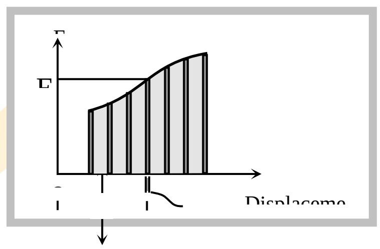
Absolute units	Gravitational units

Joule [S.I.]: Work done is said to be one *Joule*, when 1 *Newton* force displaces the body through 1 *meter* in its own direction. From $W = F.s$ 1 *Joule* = 1 *Newton* . 1 *metre*	*kg-m* [S.I.]: 1 *Kg-m* of work is done when a force of 1*kg-wt.* displaces the body through 1*m* in its own direction. From $W = F\,s$ 1 *kg-m* = 1 *kg-wt* . 1 *metre* = 9.81 *N* . 1 *metre* = 9.81 *Joule*
Erg [C.G.S.] : Work done is said to be one *erg* when 1 *dyne* force displaces the body through 1 *cm* in its own direction. From $W = F\,s$ $1\,Erg = 1 Dyne \times 1 cm$ **Relation between Joule and erg** 1 *Joule* = 1 *N* . 1 *m* = 10^5 *dyne* . 10^2 *cm* = 10^7 *dyne* . *cm* = 10^7 *Erg*	*gm-cm* [C.G.S.] : 1 *gm-cm* of work is done when a force of 1*gm-wt* displaces the body through 1*cm* in its own direction. From $W = F\,s$ 1 *gm-cm* = 1*gm-wt* . 1*cm.* = 981 *dyne* . 1*cm* = 981 *erg*

6.6 Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.

Let $\vec{F}$ be the average value of variable force within the interval *dx* from position *x* to (*x* + *dx*) *i.e.* for small displacement *dx*. The work done will be the area of the shaded strip of width *dx*. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips



$$dW = \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F}\,dx$$

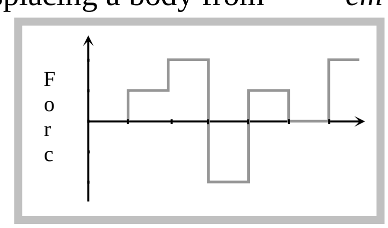
$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve Between } x_i \text{ and } x_f$$

i.e. Area under force displacement curve with proper algebraic sign represents work done by the force.

Problems based on force displacement graph

Problem 6. The relationship between force and position is shown in the figure given (in one dimensional case). The work done by the force in displacing a body from $x = 1$ *cm* to $x = 5$ *cm* is



(a) 20 *ergs*

(b) 60 *ergs*

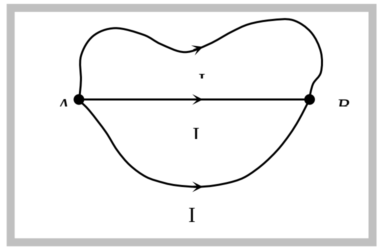
(c) 70 *ergs* (d) 700 *ergs*

Solution : (a) Work done = Covered area on force-displacement graph

$$= 1 \cdot 10 + 1 \cdot 20 - 1 \cdot 20 + 1 \cdot 10 = 20 \; erg.$$

6.7 Work Done in Conservative and Non-Conservative Field

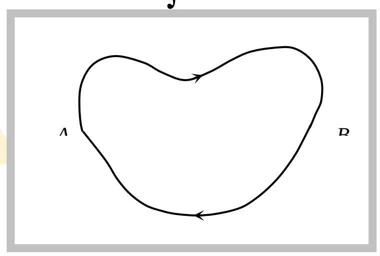
(1) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) is independent of the path followed between any two points.



$$\underset{\text{Path I}}{W_{A\rightarrow B}} = \underset{\text{Path II}}{W_{A\rightarrow B}} = \underset{\text{Path III}}{W_{A\rightarrow B}}$$

or $$\underset{\text{Path I}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path II}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path III}}{\int \vec{F}.d\vec{l}}$$

(2) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) over a closed path/loop is zero.



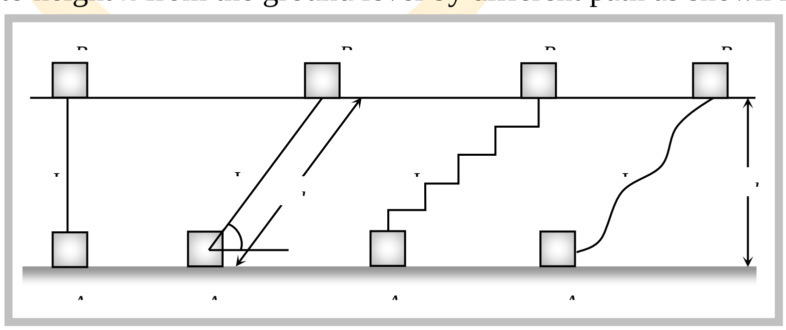
$$W_{A\rightarrow B} + W_{B\rightarrow A} = 0$$

or $$\oint \vec{F}.d\vec{l} = 0$$

Conservative force : The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of man *m* lifted to height *h* from the ground level by different path as shown in the figure



Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Airdrag *etc*.

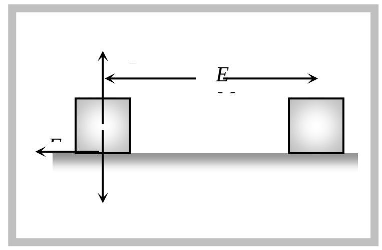
If a body is moved from position *A* to another position *B* on a rough table, work done against frictional

force shall depends on the length of the path between *A* and *B* and not only on the position *A* and *B.* $W_{AB} = \mu mgs$

Further if the body is brought back to its initial position *A*, work has to be done against the frictional force,

which always opposes the motion. Hence the net work done against the

friction over a round trip is not zero.



$$W_{BA} = \mu mgs.$$

$$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$$

i.e. the friction is a non-conservative force.

Problems based on work done in conservative and non-conservative field

***<u>Problem</u>* 7.** A particle of mass 0.01 *kg* travels along a curve with velocity given by $4\hat{i} + 16\hat{k}$ ms^{-1}. After some time, its velocity becomes $8\hat{i} + 20\hat{j}\ ms^{-1}$ due to the action of a conservative force. The work done on particle during this interval of time is

(a) 0.32 *J* (b) 6.9 *J* (c) 9.6 *J* (d) 0.96 *J*

Solution : (d) $v_1 = \sqrt{4^2 + 16^2} = \sqrt{272}$ and $v_2 = \sqrt{8^2 + 20^2} = \sqrt{464}$

Work done = Increase in kinetic energy $= \frac{1}{2}m[v_2^2 - v_1^2] = \frac{1}{2} \times 0.01[464 - 272] = 0.96\,J$

NCERT 6.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

NCERT 6.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the (a) work done by the applied force in 10 s, (b) work done by friction in 10 s, (c) work done by the net force on the body in 10 s, (d) change in kinetic energy of the body in 10 s, and interpret your results.

NCERT 6.5 Answer the following :

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?

NCERT 6.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = k x^2/2$, where k is the force constant of the oscillator. For $k = 0.5$ N m^{-1}, the graph of $V(x)$ versus x is shown in Fig. 6.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.

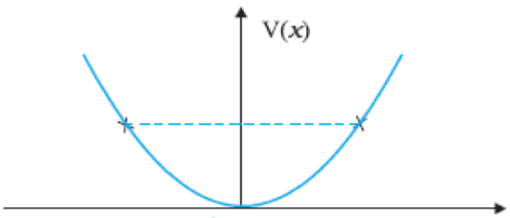


Fig. 6.12

NCERT 6.9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

NCERT 6.10 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t1/2$ (ii) t (iii) $t3/2$ (iv) $t2$

NCERT.6.11 A body constrained to move along the z-axis of a coordinate system is subject to a constant force **F** given by $\mathbf{F} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ N

where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors along the x-, y- and z-axis of the system respectively.

What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

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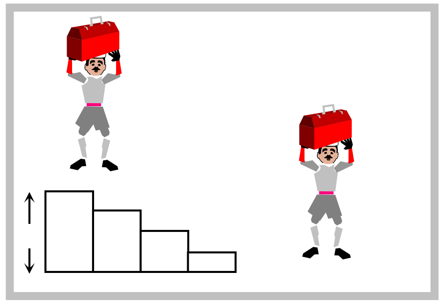
6.8 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.



Examples : (1) If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be *mgh*.

(2) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s}+\vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

6.9 Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done therefore it is a scalar quantity.

(2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : *Joule* [S.I.], *erg* [C.G.S.]

Practical units : *electron volt* (*eV*), Kilowatt hour (*KWh*), Calories (*Cal*)

Relation between different units: 1 *Joule* = 10^7 *erg*

1 *eV* = 1.6×10^{-19} *Joule*

1 *KWh* = 3.6×10^{6} *Joule*

1 *Calorie* = 4.18 *Joule*

(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential) (ii) Chemical energy (iii) Electrical energy

(iv) Magnetic energy (v) Nuclear energy (vi) Sound energy

(vii) Light energy (viii) Heat energy

Problems based on energy

<u>Problem</u> 8. A metallic wire of length *L metres* extends by *l metres* when stretched by suspending a weight *Mg* to it. The mechanical energy stored in the wire is

(a) $2Mgl$ (b) Mgl (c) $\frac{Mgl}{2}$ (d) $\frac{Mgl}{4}$

Solution : (c) Elastic potential energy stored in wire $U=\frac{1}{2}Fx=\frac{Mgl}{2}$.

6.10 Kinetic Energy

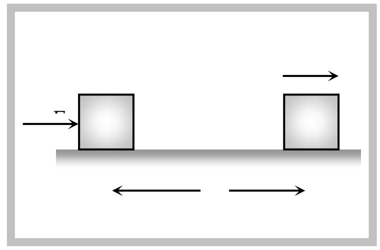
The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.
(iii) Moving air (*i.e.* wind) possesses kinetic energy which is used to run wind mills.
(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.
(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

(1) **Expression for kinetic energy :** Let

m = mass of the body, u = Initial velocity of the body (= 0)
F = Force acting on the body, a = Acceleration of the body
s = Distance travelled by the body, v = Final velocity of the body



From $v^2 = u^2 + 2as$

® $v^2 = 0 + 2as$ $\therefore s = \frac{v^2}{2a}$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$$W = F \times s = ma \times \frac{v^2}{2a}$$

$$\Rightarrow W = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = W = \frac{1}{2}mv^2$

(2) **Calculus method :** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through $d\bar{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

® $dW = m\,a\,ds$ [As $F = ma$]

® $dW = m\frac{dv}{dt}ds$ $\left[As\ a = \frac{dv}{dt}\right]$

® $dW = mdv.\frac{ds}{dt}$

® $dW = m\,v\,dv$ ....(i) $\left[As\ \frac{ds}{dt} = v\right]$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = \frac{1}{2}mv^2$.

In vector form $KE = \frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar *i.e.* kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Work-energy theorem:** From equation (i) $dW = mv\,dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

® $W = \frac{1}{2}m[v^2 - u^2]$

Work done = change in kinetic energy

$$W = \Delta E$$

This is work energy theorem, it states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive *i.e.* body moves in the direction of the force (or field) and if kinetic energy decreases work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P = mv]$$

4 $$E = \frac{1}{2}Pv$$

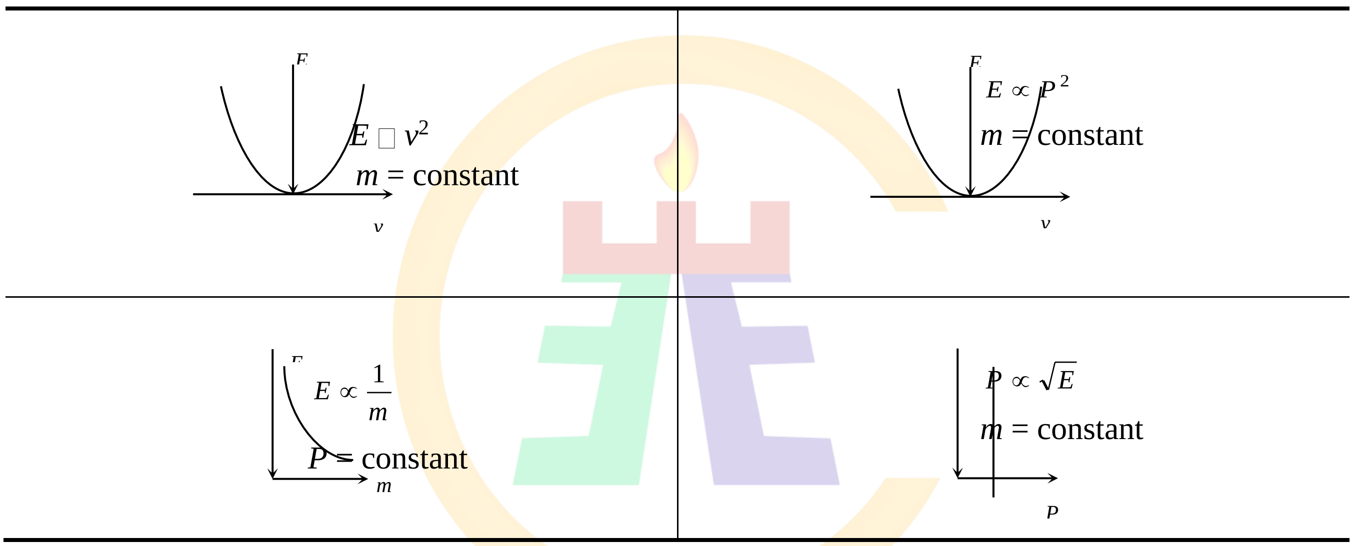
or $$E = \frac{P^2}{2m} \qquad \left[\text{As } v = \frac{P}{m}\right]$$

So we can say that kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2}Pv = \frac{p^2}{2m}$

and Momentum $P = \frac{2E}{v} = \sqrt{2mE}$.

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) **Various graphs of kinetic energy**



Sample problem based on kinetic energy

***<u>Problem</u>* 9.** Consider the following two statements

1. Linear momentum of a system of particles is zero

2. Kinetic energy of a system of particles is zero

Then

(a) 1 implies 2 and 2 implies 1 (b) 1 does not imply 2 and 2 does not imply 1

(c) 1 implies 2 but 2 does not imply 1 (d) 1 does not imply 2 but 2 implies 1

Solution : (d) Momentum is a vector quantity whereas kinetic energy is a scalar quantity. If the kinetic energy of a system is zero then linear momentum definitely will be zero but if the momentum of a system is zero then kinetic energy may or may not be zero.

***<u>Problem</u>* 10.** If the momentum of a body is increased by 100 %, then the percentage increase in the kinetic energy is

(a) 150 % (b) 200 % (c) 225 % (d) 300 %

Solution : (d) $E = \frac{P^2}{2m}$ ® $\frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{2P}{P}\right)^2 = 4$

$E_2 = 4E_1 = E_1 + 3E_1 = E_1 + 300$ % of E_1.

Problem **11.** Two masses of 1*g* and 9*g* are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is

(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1

Solution : (c) $P = \sqrt{2mE}$ 4 $P \propto \sqrt{m}$ if E = constant . So $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$.

Problem **12.** A 300 *g* mass has a velocity of $(3\hat{i} + 4\hat{j})$ *m/sec* at a certain instant. What is its kinetic energy

(a) 1.35 *J* (b) 2.4 *J* (c) 3.75 *J* (d) 7.35 *J*

Solution : (c) $\vec{v} = (3\hat{i} + 4\hat{j})$ 4 $v = \sqrt{3^2 + 4^2} = 5\, m/s$. So kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times (5)^2 = 3.75\ J$

6.11 Stopping of Vehicle by Retarding Force

If a vehicle moves with some initial velocity and due to some retarding force it stops after covering some distance after some time.

(1) **Stopping distance :** Let m = Mass of vehicle, v = Velocity, P = Momentum, E = Kinetic energy

F = Stopping force, x = Stopping distance, t = Stopping time

Then, in this process stopping force does work on the vehicle and destroy the motion.

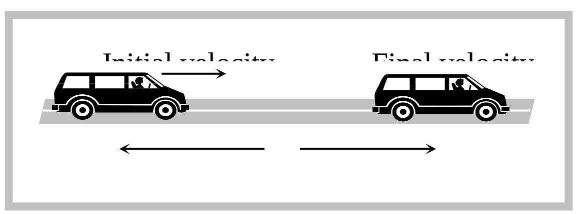
By the work- energy theorem

$$W = \Delta K = \frac{1}{2}mv^2$$

® Stopping force (F) . Distance (x) = Kinetic energy (E)

® Stopping distance (x) $= \frac{\text{Kinetic energy } (E)}{\text{Stopping force } (F)}$

® $x = \frac{mv^2}{2F}$ …..(i)



(2) **Stopping time :** By the impulse-momentum theorem

$$F \times t = \Delta P \Rightarrow F \times t = P$$

4 $t = \frac{P}{F}$

or $t = \frac{mv}{F}$ …..(ii)

(3) **Comparison of stopping distance and time for two vehicles :** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\frac{x_1}{x_2} = \frac{E_1}{E_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$

and the ratio of their stopping time $\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{m_1 v_1}{m_2 v_2}$

If vehicles possess same velocities		
$v_1 = v_2$	$\frac{x_1}{x_2} = \frac{m_1}{m_2}$	$\frac{t_1}{t_2} = \frac{m_1}{m_2}$
If vehicle possess same kinetic momentum		
$P_1 = P_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = \left(\frac{P_1^2}{2m_1}\right)\left(\frac{2m_2}{P_2^2}\right) = \frac{m_2}{m_1}$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$
If vehicle possess same kinetic energy		
$E_1 = E_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = 1$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2m_1 E_1}}{\sqrt{2m_2 E_2}} = \sqrt{\frac{m_1}{m_2}}$

Note : ✈ If vehicle is stopped by friction then

Stopping distance $x = \frac{\frac{1}{2}mv^2}{F} = \frac{\frac{1}{2}mv^2}{ma} = \frac{v^2}{2\mu g}$ [As $a = \mu g$]

Stopping time $t = \frac{mv}{F} = \frac{mv}{m\mu g} = \frac{v}{\mu g}$

Problems based on stopping of vehicle

Problem **13.** A bus can be stopped by applying a retarding force F when it is moving with a speed v on a level road. The distance covered by it before coming to rest is s. If the load of the bus increases by 50 % because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be

(a) 1.5 s (b) 2 s (c) 1 s (d) 2.5 s

Solution : (a) Retarding force (F) . distance covered (x) = Kinetic energy $\left(\frac{1}{2}mv^2\right)$

If v and F are constants then $x \propto m$ 4 $\frac{x_2}{x_1} = \frac{m_2}{m_1} = \frac{1.5m}{m} = 1.5$ ® $x_2 = 1.5\ s$.

Problem **14.** A vehicle is moving on a rough horizontal road with velocity v. The stopping distance will be directly proportional to

(a) $\sqrt{v}$ (b) v (c) v^2 (d) v^3

Solution : (c) As $s = \frac{v^2}{2a}$ 4 $s \propto v^2$.

6.12 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy *etc.*

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad \text{......(i)}$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there, *i.e.* if take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from reference position to given position.

This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive *i.e.* potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative *i.e.* potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

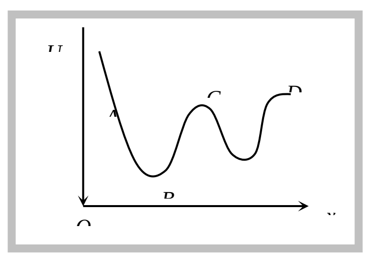
So $\vec{F} = -\vec{\nabla}U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$)

® $\vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]$

where $\frac{dU}{dx} =$ Partial derivative of U *w.r.t.* x (keeping y and z constant)

$\frac{dU}{dy} =$ Partial derivative of U *w.r.t.* y (keeping x and z constant)

$\frac{dU}{dz} =$ Partial derivative of U *w.r.t.* z (keeping x and y constant)



(3) **Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.
As we know that negative gradient of the potential energy gives force.

4 $-\frac{dU}{dx} = F$

(4) **Nature of force :**

(i) Attractive force : On increasing x, if U increases $\frac{dU}{dx}$ = positive

then F is negative in direction *i.e.* force is attractive in nature. In graph this is represented in region BC.

(ii) Repulsive force : On increasing x, if U decreases $\frac{dU}{dx}$ = negative

then F is positive in direction *i.e.* force is repulsive in nature. In graph this is represented in region AB.

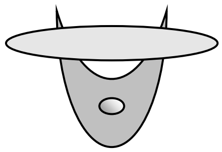
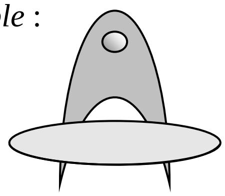
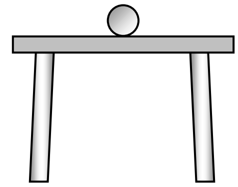
(iii) Zero force : On increasing x, if U does not changes $\frac{dU}{dx} = 0$

then F is zero *i.e.* no force works on the particle. Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

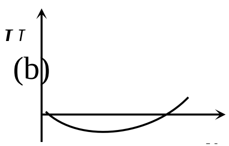
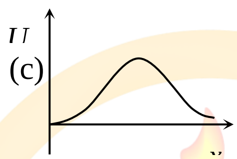
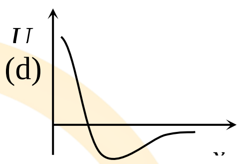
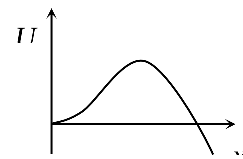
(5) **Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

Stable	Unstable	Neutral
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2}$ = positive *i.e.* rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2}$ = negative *i.e.* rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ *i.e.* rate of change of $\frac{dU}{dx}$ is zero.

Example :  A marble placed at the bottom of a hemispherical bowl.	*Example* :  A marble balanced on top of a hemispherical bowl.	*Example* :  A marble placed on horizontal table.

***Problem* 15.** A particle which is constrained to move along the *x*-axis, is subjected to a force in the same direction which varies with the distance *x* of the particle from the origin as $F(x)=-kx+ax^3$. Here *k* and *a* are positive constants. For $x\geq 0$, the functional from of the potential energy $U_{(x)}$ of the particle is

(a) 
(b) 
(c) 
(d) 

Solution : (d) $F=-\frac{dU}{dx}$ ® $dU=-F.dx$ ® $U=-\int_0^x(-kx+ax^3)dx$ ® $U=\frac{kx^2}{2}-\frac{ax^4}{4}$

4 We get $U=0$ at $x = 0$ and $x=\sqrt{\frac{2k}{a}}$

Also we get $U=$ negative for $x>\sqrt{\frac{2k}{a}}$

From the given function we can see that $F = 0$ at $x = 0$ *i.e.* slope of *U*-*x* graph is zero at $x = 0$.

***Problem* 16.** The potential energy of a body is given by $A-Bx^2$ (where *x* is the displacement). The magnitude of force acting on the particle is **[BHU 2002]**

(a) Constant
(b) Proportional to *x*
(c) Proportional to x^2
(d) Inversely proportional to *x*

Solution : (b) $F=\frac{-dU}{dx}=-\frac{d}{dx}(A-Bx^2)=2Bx$ 4 F □ x.

***Problem* 17.** A particle moves in a potential region given by $U=8x^2-4x+400$ *J*. Its state of equilibrium will be

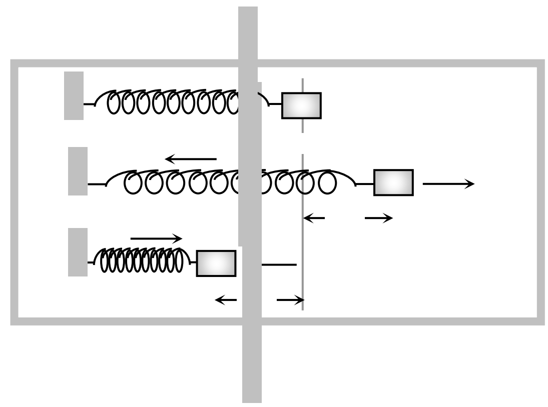
(a) $x=25\,m$
(b) $x=0.25\,m$
(c) $x=0.025\,m$
(d) $x=2.5\,m$

Solution : (b) $F=-\frac{dU}{dx}=-\frac{d}{dx}(8x^2-4x+400)$

For the equilibrium condition $F=-\frac{dU}{dx}=0$ ® $16x-4=0$ ® $x=4/16$ 4 $x=0.25\,m$.

6.13 Elastic Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position (*x* = 0) by a small distance *x*, then a restoring force is produced in the spring to bring it to the normal position.



According to Hooke's law this restoring force is proportional to the displacement x and its direction is

always opposite to the displacement.

i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\vec{x}$ …..(i)

where k is called spring constant.

If $x = 1$, $F = k$ (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$ 4 $[k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit *Newton*/*metre*, C.G.S unit *Dyne*/*cm*.

Note : ✈ Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position (x = 0), work has to be done by external force against restoring force. $\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.d\vec{x} = F_{ext}.dx\cos 0^o = kx\,dx \qquad [\text{As } \cos 0^o = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

4 Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

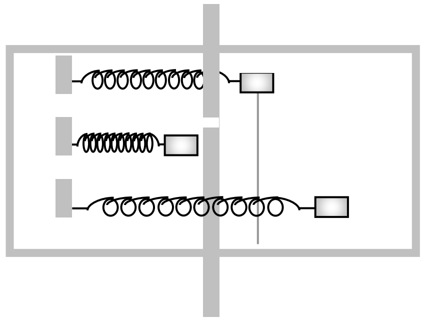
$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

4 Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$U = \frac{1}{2}kx^2$ ….(i)

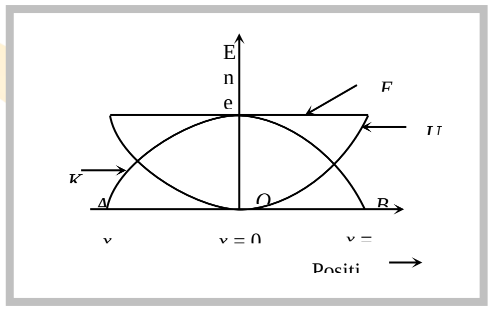
So for the extreme position

$U = \frac{1}{2}ka^2$ [As $x = \pm a$ for extreme]

This is maximum potential energy or the total energy of mass.

4 Total energy $E = \frac{1}{2}ka^2$ ….(ii)

[Because velocity of mass = 0 at extreme 4 $K = \frac{1}{2}mv^2 = 0$]



Now kinetic energy at any position $K = E - U$ $= \frac{1}{2}k\,a^2 - \frac{1}{2}k\,x^2$

$K = \frac{1}{2}k(a^2 - x^2)$ ……(iii)

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme $x = \pm a$] and $U_{min} = 0$ [At mean $x = 0$]

$K_{max} = \frac{1}{2}ka^2$ [At mean $x = 0$] and $K_{min} = 0$ [At extreme $x = \pm a$]

$E = \frac{1}{2}ka^2 =$ constant (at all positions)

It mean kinetic energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass

Problems based on elastic potential energy

<u>Problem</u> **18.** Two springs of spring constants $1500\ N/m$ and $3000\ N/m$ respectively are stretched with the same force. They will have potential energy in the ratio

(a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

Solution : (c) Potential energy of spring $U = \frac{F^2}{2k}$ ® $\frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = 2:1$ [If F = constant]

<u>Problem</u> **19.** Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

Solution : (d) If the spring is stretched by length *x*, then work done by two equal masses = $\frac{1}{2}kx^2$

So work done by each mass on the spring = $\frac{1}{4}kx^2$ 4 Work done by spring on each mass = $-\frac{1}{4}kx^2$.

NCERT 6.17 The bob A of a pendulum released from 30º to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6.15. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.

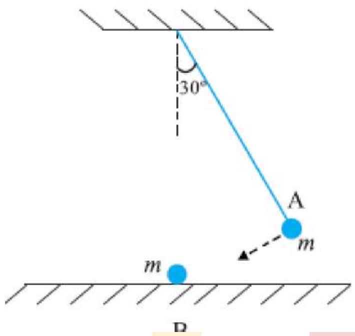


Fig. 6.15

NCERT 6.18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance ?

NCERT 6.19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

NCERT 6.20 A body of mass 0.5 kg travels in a straight line with velocity *v* =*a x*3/2 where *a* = 5 m–1/2 s^{-1}. What is the work done by the net force during its displacement from *x* = 0 to *x* = 2 m ?

NCERT 6.21 The blades of a windmill sweep out a circle of area *A*. (a) If the wind flows at a velocity *v* perpendicular to the circle, what is the mass of the air passing through it in time *t* ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, *v* = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced ?

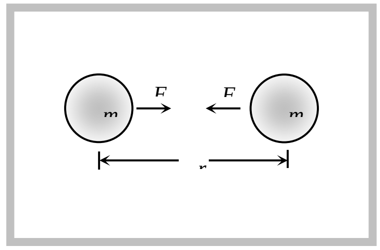
NCERT 6.22 A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

NCERT 6.23 A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal

surface at an average rate of 200 W per square meter. If 20% of this energy can be Converted to useful electrical energy, how large an area is needed to supply 8 kW?
(b) Compare this area to that of the roof of a typical house.

6.15 Gravitational Potential Energy

It is the usual form of potential energy and is the energy associated with the state of separation between two bodies that interact via gravitational force.



For two particles of masses m_1 and m_2 separated by a distance r

Gravitational potential energy $U = -\frac{Gm_1m_2}{r}$

(1) If a body of mass m at height h relative to surface of earth then

Gravitational potential energy $U = \frac{mgh}{1 + \frac{h}{R}}$

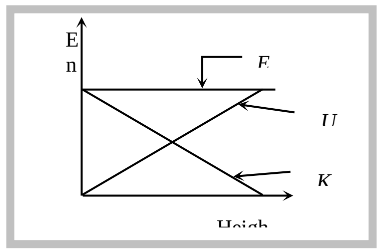
Where R = radius of earth, g = acceleration due to gravity at the surface of the earth.

(2) If $h << R$ then above formula reduces to $U = mgh$.

(3) If V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be

$$U = mV$$

(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its potential energy is zero.



As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion total energy remains constant as shown in the figure.

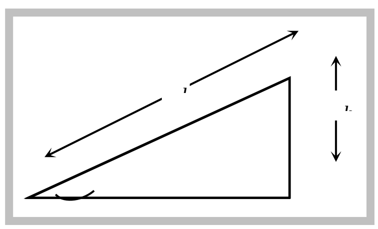
Sample problems based on gravitational potential energy

<u>Problem</u> 20. The work done in pulling up a block of wood weighing 2*kN* for a length of 10 *m* on a smooth plane inclined at an angle of 15^o with the horizontal is (sin 15° = 0.259)

[AFMC 1999]

(a) 4.36 *k J* (b) 5.17 *k J* (c) 8.91 *k J* (d) 9.82 *k J*

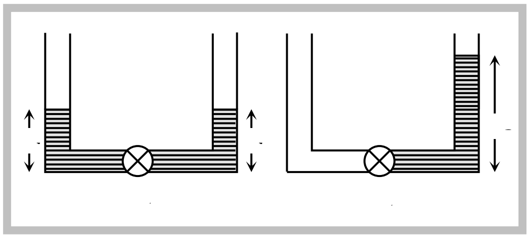
Solution : (b) Work done = $mg \cdot h$



$= 2 \times 10^3 \times l \sin\theta$

$= 2 \times 10^3 \times 10 \times \sin 15^o = 5176\ J = 5.17\ kJ$

Problem **21.** A liquid of density d is pumped by a pump P from situation (i) to situation (ii) as shown in the diagram. If the cross-section of each of the vessels is a, then the work done in pumping (neglecting friction effects) is



(a) $2dgh$
(b) $dgha$
(c) $2dgh^2a$
(d) dgh^2a

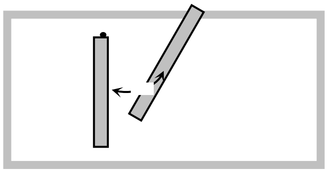
Solution : (d) Potential energy of liquid column in first situation $= Vdg\frac{h}{2} + Vdg\frac{h}{2} = Vdgh = ahdgh = dgh^2a$

[As centre of mass of liquid column lies at height $\frac{h}{2}$]

Potential energy of the liquid column in second situation $= Vdg\left(\frac{2h}{2}\right) = (A\times 2h)dgh = 2dgh^2a$

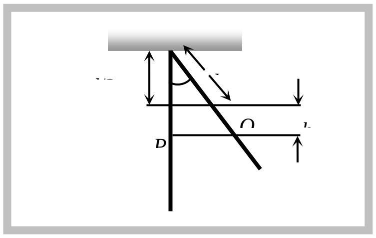
Work done pumping = Change in potential energy = $2dgh^2a - dgh^2a = dgh^2a$.

Problem **22.** A *metre* stick, of mass 400 *g*, is pivoted at one end displaced through an angle 60^o. The increase in its potential energy is



(a) 1 *J*
(b) 10 *J*
(c) 100 *J*
(d) 1000 *J*

Solution : (a) Centre of mass of a stick lies at the mid point and when the stick is displaced through an angle 60° it rises upto height '*h*' from the initial position.



From the figure $h = \frac{l}{2} - \frac{l}{2}\cos\theta = \frac{l}{2}(1-\cos\theta)$

Hence the increment in potential energy of the stick = *mgh*

$$= mg\frac{l}{2}(1-\cos\theta) = 0.4\times 10\times\frac{1}{2}(1-\cos 60^o) = 1\,J$$

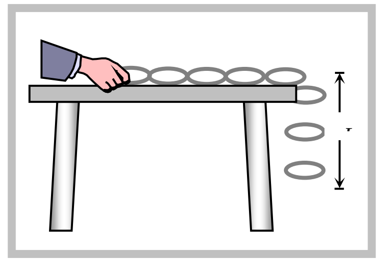
Problem **23.** Once a choice is made regarding zero potential energy reference state, the changes in potential energy

(a) Are same
(b) Are different
(c) Depend strictly on the choice of the zero of potential energy
(d) Become indeterminate

Solution : (a) Potential energy is a relative term but the difference in potential energy is absolute term. If reference level is fixed once then change in potential energy are same always.

6.16 Work Done in Pulling the Chain Against Gravity

A chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.



Let $m = \frac{M}{L} =$ mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$dW = F(-dy)$ [As y is decreasing]

i.e. $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

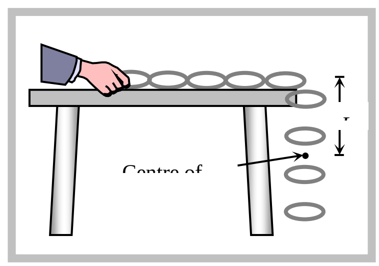
$$W = -\int_{L/n}^{0} mgy\, dy = mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mg\,L^2}{2n^2}$$

4 $W = \frac{MgL}{2n^2}$ [As $m = M/L$]

Alternative method :

If point mass m is pulled through a height h then work done $W = mgh$

Similarly for a chain we can consider its centre of mass at the middle point of the hanging part *i.e.* at a height of $L/(2n)$ from the lower end and mass of the hanging part of chain $= \frac{M}{n}$



So work done to raise the centre of mass of the chain on the table is given by

$W = \frac{M}{n} \times g \times \frac{L}{2n}$ [As $W = mgh$]

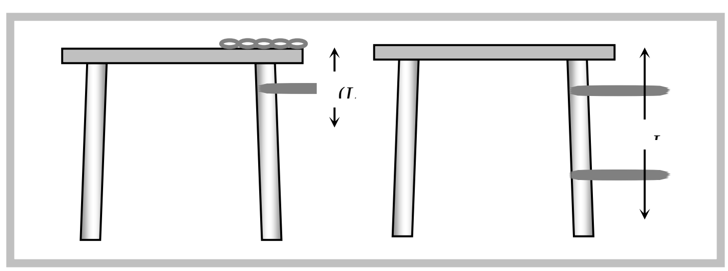
or $W = \frac{MgL}{2n^2}$

6.17 Velocity of Chain While Leaving the Table

Taking surface of table as a reference level (zero potential energy)

Potential energy of chain when $1/n^{th}$ length hanging from the edge $= \frac{-MgL}{2n^2}$

Potential energy of chain when it leaves the table $= -\frac{MgL}{2}$



Kinetic energy of chain = loss in potential energy

® $\frac{1}{2}Mv^2 = \frac{MgL}{2} - \frac{MgL}{2n^2}$

® $\frac{1}{2}Mv^2 = \frac{MgL}{2}\left[1 - \frac{1}{n^2}\right]$

4 Velocity of chain $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}$

Problem based on chain

***Problem* 24.** A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is **[IIT-JEE 1985; MNR 1990; MP PMT 1994, 97, 2000; JIMPER 2000; AIEEE 2002]**

(a) MgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

Solution : (d) As 1/3 part of the chain is hanging from the edge of the table. So by substituting $n = 3$ in standard expression

$$W = \frac{MgL}{2n^2} = \frac{MgL}{2(3)^2} = \frac{MgL}{18}.$$

***Problem* 25.** A chain is placed on a frictionless table with one fourth of it hanging over the edge. If the length of the chain is $2m$ and its mass is $4kg$, the energy need to be spent to pull it back to the table is

(a) 32 J (b) 16 J (c) 10 J (d) 2.5 J

Solution : (d) $W = \frac{MgL}{2n^2} = \frac{4 \times 10 \times 2}{2 \times (4)^2} = 2.5\ J.$

6.18 Law of Conservation of Energy

(1) **Law of conservation of energy**

For a body or an isolated system by work-energy theorem we have $K_2 - K_1 = \int \vec{F}.d\vec{r}$ .....(i)

But according to definition of potential energy in a conservative field $U_2 - U_1 = -\int \vec{F}.d\vec{r}$ .....(ii)

So from equation (i) and (ii) we have

$$K_2 - K_1 = -(U_2 - U_1)$$

or $K_2 + U_2 = K_1 + U_1$

i.e. $K + U =$ constant.

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

$\Delta(K + U) = \Delta E = 0$ [As E is constant in a conservative field]

4 $\Delta K + \Delta U = 0$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount of work done by the frictional force.

$$\Delta(K+U) = \Delta E = W_f \qquad \text{[where } W_f \text{ is the work done against friction]}$$

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

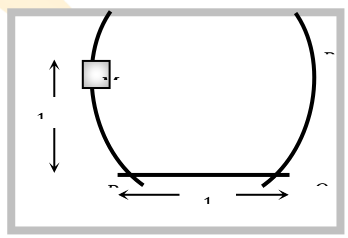
We can, therefore, write ⊗$E + Q = 0$ [where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc*., which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant*". This is the law of conservation of energy.

Problems based on conservation of energy

Problem **26.** A block of mass M slides along the sides of a bowl as shown in the figure. The walls of the bowl are frictionless and the base has coefficient of friction 0.2. If the block is released from the top of the side, which is 1.5 m high, where will the block come to rest ? Given that the length of the base is 15 m



(a) 1 m from P

(b) Mid point

(c) 2 m from P

(d) At Q

Solution : (b) Potential energy of block at starting point = Kinetic energy at point P = Work done against friction in traveling a distance s from point P.

4 $mgh = /mgs$ ® $s = \frac{h}{\mu} = \frac{1.5}{0.2} = 7.5\,m$

i.e. block come to rest at the mid point between P and Q.

Problem **27.** A 2kg block is dropped from a height of 0.4 m on a spring of force constant $K = 1960\ Nm^{-1}$. The maximum compression of the spring is

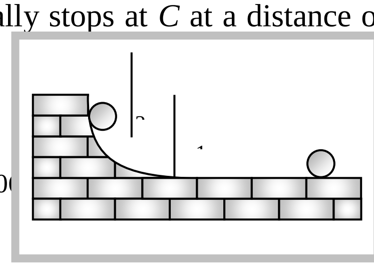
(a) 0.1 m (b) 0.2 m (c) 0.3 m (d) 0.4 m

Solution : (a) When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring get compressed due to this energy.

4 Gravitational potential energy of block = Elastic potential energy of spring

® $mgh = \frac{1}{2}Kx^2$ ® $x = \sqrt{\frac{2\,mgh}{K}} = \sqrt{\frac{2\times 2\times 10\times 0.4}{1960}} = 0.09\,m \simeq 0.1\,m$.

Problem **28.** A block of mass 2kg is released from A on the track that is one quadrant of a circle of radius 1m. It slides down the track and reaches B with a speed of $4\,ms^{-1}$ and finally stops at C at a distance of 3m from B. The work done against the force of friction is



(a) 10 J

(b) 20 J

(c) 2 J

(d) 6 J

Solution : (b) Block possess potential energy at point A = mgh $= 2 \times 10 \times 1 = 20\ J$

Finally block stops at point C. So its total energy goes against friction *i.e.* work done against friction is 20 J.

Problem **29.** A stone projected vertically upwards from the ground reaches a maximum height h. When it is at a height $\frac{3h}{4}$, the ratio of its kinetic and potential energies is

(a) 3 : 4 (b) 1 : 3 (c) 4 : 3 (d) 3 : 1

Solution : (b) At the maximum height, Total energy = Potential energy = mgh

At the height $\frac{3h}{4}$, Potential energy = $mg\frac{3h}{4} = \frac{3}{4}mgh$

and Kinetic energy = Total energy – Potential energy $= mgh - 3\frac{mgh}{4} = \frac{1}{4}mgh$

4 $\frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{3}$.

6.19 Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Important points

(1) Dimension : $[P] = [F][v] = [MLT^{-2}][LT^{-1}]$

4 $[P] = [ML^2T^{-3}]$

(2) Units : *Watt* or *Joule*/*sec* [S.I.]

Erg/*sec* [C.G.S.]

Practical units : *Kilowatt* (*kW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units : $1\,watt = 1\,Joule/\sec = 10^7\,erg/\sec$

$$1hp = 746\ Watt$$

$$1\ MW = 10^6\ Watt$$

$$1\ kW = 10^3\ Watt$$

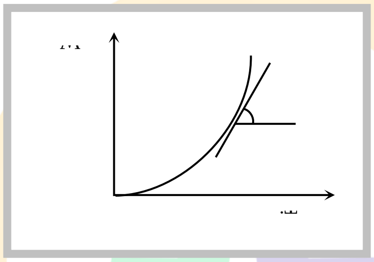
(3) If work done by the two bodies is same then power $\propto \frac{1}{\text{time}}$

i.e. the body which perform the given work in lesser time possess more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\ KWh = 10^3\ \frac{J}{sec} \times (60 \times 60\ sec) = 3.6 \times 10^6\ Joule$$

(5) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan($



(6) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

4 $W = \int P\, dt$

4 W = Area under P-t curve

6.20 Position and Velocity of an Automobile *w.r.t* Time

An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and velocity changes *w.r.t* time.

(1) **Velocity :** As $Fv = P$ = constant

i.e. $m\frac{dv}{dt}v = P$ $\left[\text{As } F = \frac{mdv}{dt}\right]$

or $\int v\, dv = \int \frac{P}{m} dt$

By integrating both sides we get $\frac{v^2}{2} = \frac{P}{m}t + C_1$

As initially the body is at rest *i.e.* $v = 0$ at $t = 0$, so $C_1 = 0$

4 $v = \left(\frac{2Pt}{m}\right)^{1/2}$

(2) **Position :** From the above expression $v = \left(\frac{2Pt}{m}\right)^{1/2}$

or $\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$ $\left[\text{As}\, v = \frac{ds}{dt}\right]$

i.e. $\int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$

By integrating both sides we get $s = \left(\frac{2P}{m}\right)^{1/2} . \frac{2}{3} t^{3/2} + C_2$

Now as at $t = 0$, $s = 0$, so $C_2 = 0$

4 $s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

Problems based on power

***Problem* 30.** A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed *v*, the electrical power output will be proportional to **[IIT-JEE 2000]**

(a) v (b) v^2 (c) v^3 (d) v^4

Solution : (c) Force $= v\frac{dm}{dt} = v\frac{d}{dt}(V \times \rho) = v\rho \frac{d}{dt}[A \times l] = v\rho A \frac{dl}{dt} = \rho A v^2$

Power = $F . v = \rho A v^2 \times v = \rho A v^3$ 4 $P \propto v^3$.

***Problem* 31.** A pump motor is used to deliver water at a certain rate from a given pipe. To obtain twice as much water from the same pipe in the same time, power of the motor has to be increased to **[JIPMER 2002]**

(a) 16 times (b) 4 times (c) 8 times (d) 2 times

Solution : (d) $P = \frac{\text{work done}}{\text{time}} = \frac{mgh}{t}$ 4 P □ m

i.e. To obtain twice water from the same pipe in the same time, the power of motor has to be increased to 2 times.

***Problem* 32.** A particle moves with a velocity $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}\, ms^{-1}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}\, N$. The instantaneous power applied to the particle is

(a) 200 $J\text{-}s^{-1}$ (b) 40 $J\text{-}s^{-1}$ (c) 140 $J\text{-}s^{-1}$ (d) 170 $J\text{-}s^{-1}$

Solution : (c) $P = \vec{F}.\vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k}) = 50 - 30 + 120 = 140\ J\text{-}s^{-1}$

***Problem* 33.** A car of mass 1250 *kg* experience a resistance of 750 *N* when it moves at 30ms^{-1}. If the engine can develop 30*kW* at this speed, the maximum acceleration that the engine can produce is

(a) $0.8 ms^{-2}$ (b) $0.2 ms^{-2}$ (c) $0.4 ms^{-1}$ (d) $0.5 ms^{-2}$

Solution : (b) Power = Force . velocity = (Resistive force + Accelerating force) . velocity

® $30\times10^3 = (750+ma)\times30$ ® $ma = 1000-750$ ® $a = \frac{250}{1250} = 0.2\,ms^{-2}$.

Problem **34.** Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is

(a) $\frac{15}{11}$ (b) $\frac{11}{15}$ (c) $\frac{11}{9}$ (d) $\frac{9}{11}$

Solution : (a) Power (P) = $\frac{mgh}{t}$ or $P\propto\frac{m}{t}$ ® $\frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{t_2}{t_1} = \left(\frac{5}{3}\right)\left(\frac{9}{11}\right) = \frac{45}{33} = \frac{15}{11}$ (g and h are constants)

Problem **35.** A constant force F is applied on a body. The power (P) generated is related to the time elapsed (t) as

(a) $P\propto t^2$ (b) $P\propto t$ (c) $P\propto\sqrt{t}$ (d) $P\propto t^{3/2}$

Solution : (b) $F = \frac{mdv}{dt}$ 4 $F\,dt = mdv$ ® $v = \frac{F}{m}t$

Now $P = F\cdot v = F\times\frac{F}{m}t = \frac{F^2t}{m}$

If force and mass are constants then P □ t.

NCERT 6.24 A bullet of mass 0.012 kg and horizontal speed 70 m s^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

NCERT 6.25 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.16). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 300$, $\theta_2 = 600$, and h = 1m, what are the speeds and times taken by the two stones ?

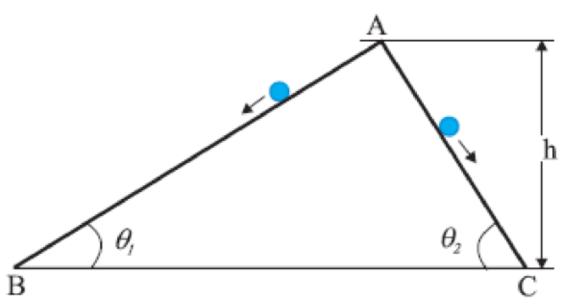


Fig. 6.16

NCERT 6.26 A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in Fig. 6.17. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.

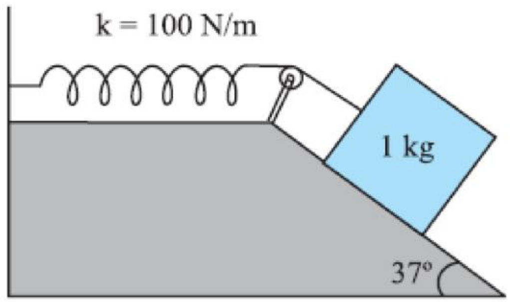


Fig. 6.17

NCERT 6.27 A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 m s^{-1}. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

NCERT 6.28 A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?

NCERT 6.29 Which of the following potential energy curves in Fig. 6.18 cannot possibly describe the elastic collision of two billiard balls ? Here *r* is the distance between centers of the balls.

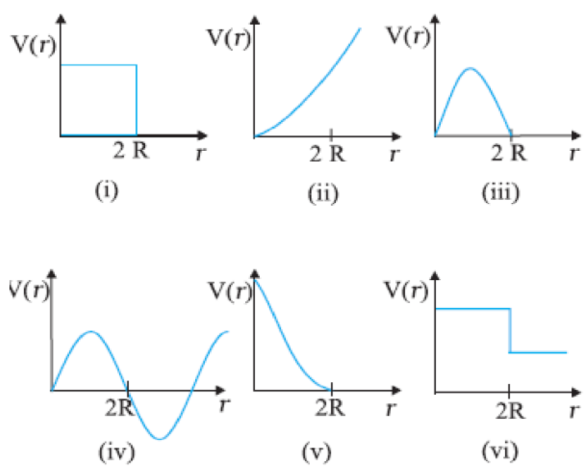


Fig. 6.18

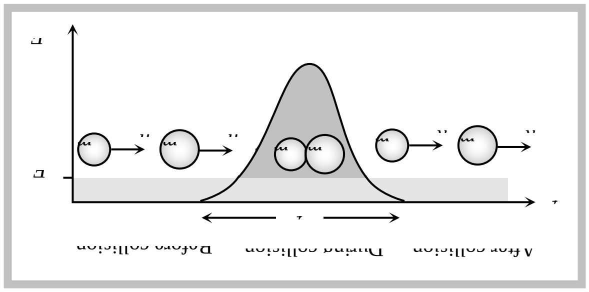
6.21 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat there is physical contact while in collision of alpha particle by a nucleus (*i.e*. Rutherford scattering experiment) there is no physical contact.

(1) **Stages of collision :** There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very

large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used, the law of conservation of momentum is useful in relating the initial and final velocities.



(2) **Momentum and energy conservation in collision :**

(i) Momentum conservation : In a collision the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (⊗t) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

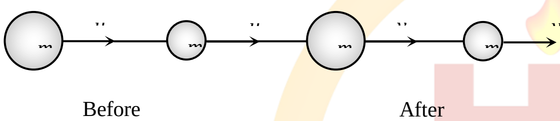
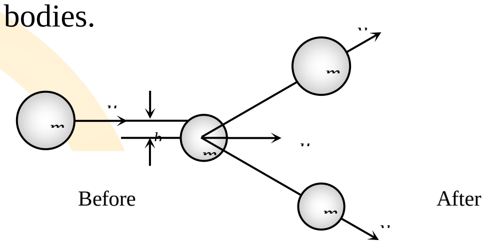
(ii) Energy conservation : In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy.

These laws are the fundamental laws of physics and applicable for any type of collision but this is not true for conservation of kinetic energy

(3) **Types of collision :** (i) On the basis of conservation of kinetic energy.

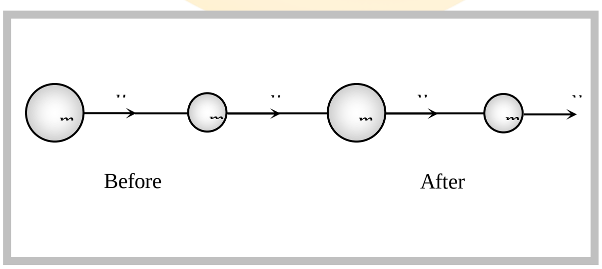
Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples* : (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	*Example* : Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	**Oblique collision**
In a collision if the motion of colliding particles before and after the collision is along the same line the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' *i.e.* such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3-dimensional.
Impact parameter b is zero for this type of collision.  	Impact parameter b lies between 0 and $(r_1 + r_2)$ *i.e.* $0 < b < (r_1 + r_2)$ where r_1 and r_2 are radii of colliding bodies. 
Example : collision of two gliders on an air track.	*Example* : Collision of billiard balls.

6.22 Perfectly Elastic Head on Collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{……(i)}$$

® $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{……(ii)}$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{…..(iii)}$$

® $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ .....(iv)

Dividing equation (iv) by equation (ii)

$v_1 + u_1 = v_2 + u_2$ .....(v)

® $u_1 - u_2 = v_2 - v_1$ .....(vi)

Relative velocity of approach = Relative velocity of separation :

✈ The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \text{or} \quad v_2 - v_1 = e(u_1 - u_2)$$

✈ For perfectly elastic collision $e = 1$ 4 $v_2 - v_1 = u_1 - u_2$ (As shown in eq. (vi)

✈ For perfectly inelastic collision $e = 0$ 4 $v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

✈ For inelastic collision $0 < e < 1$ 4 $v_2 - v_1 = e(u_1 - u_2)$

In short we can say that *e* is the degree of elasticity of collision and it is dimension less quantity.

Further from equation (v) we get $v_2 = v_1 + u_1 - u_2$

Substituting this value of v_2 in equation (i) and rearranging we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{......(vii)}$$

Similarly we get $$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2} \quad \text{......(viii)}$$

(1) **Special cases of head on elastic collision**

(i) **If projectile and target are of same mass *i.e.* $m_1 = m_2$**

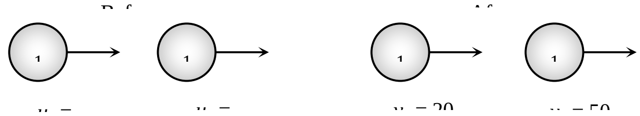
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

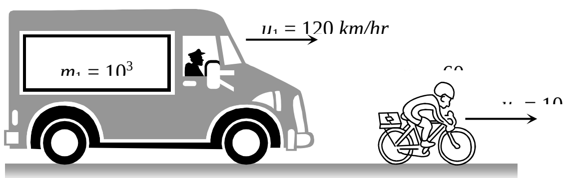
Example : Collision of two billiard balls

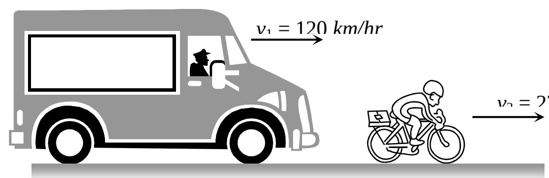


(ii) **If massive projectile collides with a light target *i.e.* $m_1 >> m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Sub case : EMBED Equation.3 i.e. target is at rest
$v_1 = u_1$ and $v_2 = 2u_1$





Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Example : Collision of a truck with a cyclist

Before collision After collision

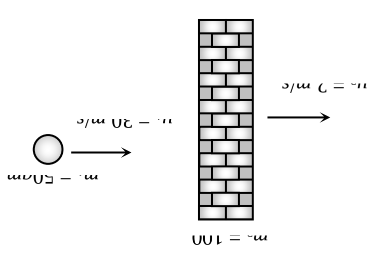
(iii) **If light projectile collides with a very heavy target *i.e.* $m_1 << m_2$**

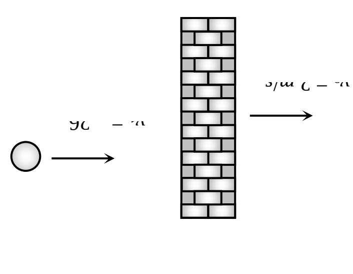
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.





Sub case : EMBED Equation.3 i.e. target is at rest

$v_1 = -u_1$ and $v_2 = 0$

i.e. the ball rebounds with same speed in opposite direction when it collide with stationary and very massive wall.

(2) **Kinetic energy transfer during head on elastic collision**

Kinetic energy of projectile before collision $K_i = \frac{1}{2}m_1u_1^2$

Kinetic energy of projectile after collision $K_f = \frac{1}{2}m_1v_1^2$

Kinetic energy transferred from projectile to target ⊗K = decrease in kinetic energy in projectile

$$\Delta K = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 \quad = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1(u_1^2 - v_1^2)}{\frac{1}{2}m_1u_1^2} = 1 - \left(\frac{v_1}{u_1}\right)^2 \quad .....(i)$$

We can substitute the value of v_1 from the equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$

If the target is at rest *i.e.* $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

From equation (i) $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ .....(ii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 + m_2)^2}$ .....(iii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2)^2 + 4m_1m_2}$ .....(iv)

Note : ✈ Greater the difference in masses less will be transfer of kinetic energy and vice versa

✈ Transfer of kinetic energy will be maximum when the difference in masses is minimum

i.e. $m_1 - m_2 = 0$ or $m_1 = m_2$ then $\frac{\Delta K}{K} = 1 = 100\%$

So the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%.

✈ If $m_2 = nm_1$ then from equation (iii) we get $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

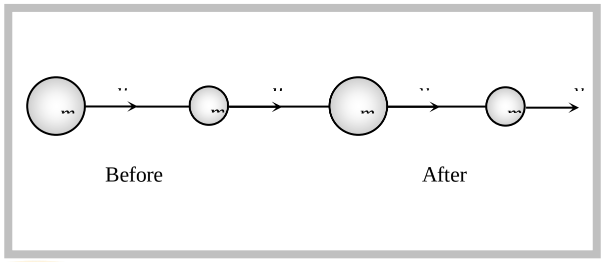
✈ Kinetic energy retained by the projectile $\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 -$ kinetic energy transferred by projectile

® $$\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1-\left[1-\left(\frac{m_1-m_2}{m_1+m_2}\right)^2\right] = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2$$

(3) **Velocity, momentum and kinetic energy of stationary target after head on elastic collision**

(i) Velocity of target :

We know $$v_2 = \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2 + \frac{2m_1u_1}{m_1+m_2}$$



® $$v_2 = \frac{2m_1u_1}{m_1+m_2} = \frac{2u_1}{1+m_2/m_1}$$

[As $u_2 = 0$ and Let $\frac{m_2}{m_1} = n$]

4 $$v_2 = \frac{2u_1}{1+n}$$

(ii) Momentum of target : $P_2 = m_2v_2 = \frac{2nm_1u_1}{1+n}$ $\left[\text{As } m_2 = m_1 n \text{ and } v_2 = \frac{2u_1}{1+n}\right]$

4 $$P_2 = \frac{2m_1u_1}{1+(1/n)}$$

(iii) Kinetic energy of target : $K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}nm_1\left(\frac{2u_1}{1+n}\right)^2 = \frac{2m_1u_1^2n}{(1+n)^2}$

$$= \frac{4(K_1)n}{(1-n)^2+4n} \qquad \left[\text{As } K_1 = \frac{1}{2}m_1u_1^2\right]$$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \frac{2u_1}{1+n}$	For v_2 to be maximum n must be minimum *i.e.* $n = \frac{m_2}{m_1} \to 0$ 4 $m_2 << m_1$	Target should be very light.
Momentum	$P_2 = \frac{2m_1u_1}{(1+1/n)}$	For P_2 to be maximum, $(1/n)$ must be minimum or n must be maximum. *i.e.* $n = \frac{m_2}{m_1} \to \infty$ 4 $m_2 >> m_1$	Target should be massive.
Kinetic energy	$K_2 = \frac{4K_1n}{(1-n)^2+4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum. *i.e.* $1-n=0 \Rightarrow n = 1 = \frac{m_2}{m_1}$ 4 $m_2 = m_1$	Target and projectile should be of equal mass.

***Problem* 36.** n small balls each of mass m impinge elastically each second on a surface with velocity u. The force experienced by the surface will be

(a) mnu (b) $2\ mnu$ (c) $4\ mnu$ (d) $\frac{1}{2}mnu$

Solution : (b)As the ball rebounds with same velocity therefore change in velocity = $2u$ and the mass colliding with the surface per second = nm Force experienced by the surface $F = m\frac{dv}{dt}$ 4 $F = 2\ mnu$.

NCERT 6.6 Underline the correct alternative :

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

NCERT 6.7 State if each of the following statements is true or false. Give reasons for your answer.

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(b) Total energy of a system is always conserved, no matter what internal and External forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

NCERT 6.8 Answer carefully, with reasons :

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact) ?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?

(c) What are the answers to (a) and (b) for an inelastic collision ?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

NCERT 6.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds. (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J).

NCERT 6.13 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its originalheight, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1}?

NCERT 6.14 A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic?

NCERT 6.15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?

NCERT 6.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *V*. If the collision is elastic, which of the following (Fig. 6.14) is a possible result after collision ?

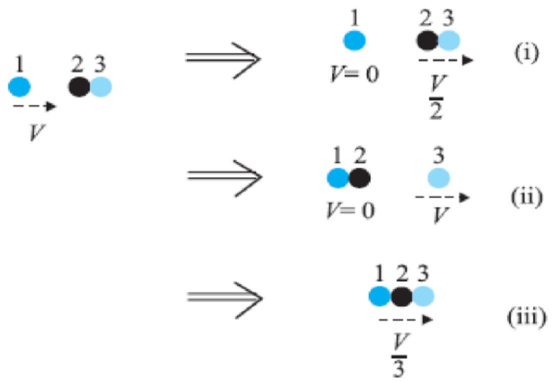


Fig. 6.14

***Problem* 36.** A body of mass *m* moving with velocity *v* makes a head-on collision with another body of mass 2*m* which is initially at rest. The loss of kinetic energy of the colliding body (mass *m*) is

(a) $\frac{1}{2}$ of its initial kinetic energy

(b) $\frac{1}{9}$ of its initial kinetic energy

(c) $\frac{8}{9}$ of its initial kinetic energy

(d) $\frac{1}{4}$ of its initial kinetic energy

Solution : (c) Loss of kinetic energy of the colliding body $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 1 - \left(\frac{m - 2m}{m + 2m}\right)^2 = 1 - \left(\frac{1}{3}\right)^2$

$\Delta K = \left(1 - \frac{1}{9}\right)K = \frac{8}{9}K$ 4 Loss of kinetic energy is $\frac{8}{9}$ of its initial kinetic energy.

***Problem* 37.** Consider the following statements

Assertion (*A*) : In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (*i.e.*, when they are in contact)

Reason (R) : Energy spent against friction does not follow the law of conservation of energy of these statements

(a) Both A and R are true and the R is a correct explanation of A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

Solution : (d) (i) When they are in contact some part of kinetic energy may convert in potential energy so it is not conserved during the short time of collision. (ii) Law of conservation of energy is always true.

Problem **38.** A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision their final velocities are V and v respectively. The value of v is **[MP PET 1995]**

(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$ (c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

Solution : (c) Final velocity of the target $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

As initially target is at rest so by substituting $u_2 = 0$ we get $v_2 = \frac{2Mu}{M+m} = \frac{2u}{1+\frac{m}{M}}$.

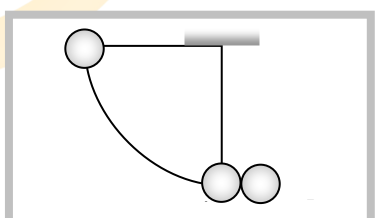
Problem **39.** A sphere of mass 0.1 kg is attached to a cord of 1m length. Starting from the height of its point of suspension this sphere hits a block of same mass at rest on a frictionless table, If the impact is elastic, then the kinetic energy of the block after the collision is **[RPET 1991]**

(a) 1 J

(b) 10 J

(c) 0.1 J

(d) 0.5 J



Solution : (a) As two blocks are of same mass and the collision their velocities gets interchanged *i.e.* the block A comes into rest and complete kinetic energy transferred to block B.

Now kinetic energy of block B after collision = Kinetic energy of block A before collision

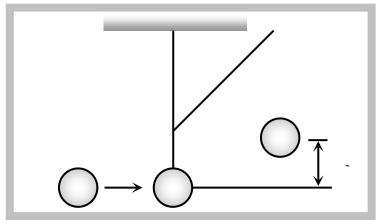
= Potential energy of block A at the original height

$= mgh = 0.1 \cdot 10 \cdot 1 = 1\ J$.

Problem **40.** A ball moving horizontally with speed v strikes the bob of a simple pendulum at rest. The mass of the bob is equal to that of the ball. If the collision is elastic the bob will rise to a height

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$ (c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

Solution : (b) Total kinetic energy of the ball will transfer to the bob of simple pendulum. Let it rises to height '*h*' by the law of conservation of energy.



$$\frac{1}{2}mv^2 = mgh$$

4 $h = \frac{v^2}{2g}$

***Problem* 41.** A body of mass *m* moving along a straight line collides with a body of mass *nm* which is also moving with a velocity *kv* in the same direction. If the first body comes to rest after the collision, then the velocity of second body after the collision would be

(a) $\frac{nv}{(1+nk)}$ (b) $\frac{nv}{(1-nk)}$ (c) $\frac{(1-nk)v}{n}$ (d) $\frac{(1+nk)v}{n}$

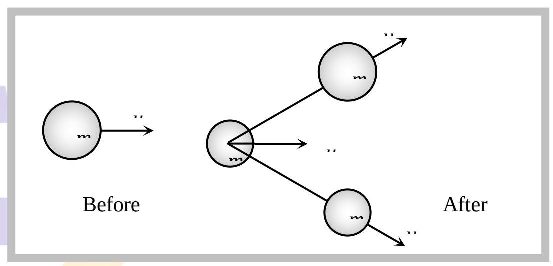
Solution : (d) Initial momentum = $mv + nm(kv)$ and final momentum $= 0 + nm\,V$

By the conservation of momentum, $mv + nm(kv) = 0 + nm\,V$

® $v + nkv = nV$ ® $nV = (1+nk)v$ ® $V = \frac{(1+nk)v}{n}$

6.23 Perfectly Elastic Oblique Collision

Let two bodies moving as shown in figure.



By law of conservation of momentum

Along *x*-axis, $m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi$ .....(i)

Along *y*-axis, $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$ .....(ii)

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad .....(iii)$$

In case of oblique collision it becomes difficult to solve problem when some experimental data are provided as in these situations more unknown variables are involved than equations formed.

Special condition : If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$$u_1 = v_1\cos\theta + v_2\cos\phi \quad .....(iv)$$

$$0 = v_1\sin\theta - v_2\sin\phi \quad .....(v)$$

and $u_1^2 = v_1^2 + v_2^2$ .....(vi)

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2\cos(\theta+\phi) \quad .....(vii)$$

Using (vi) and (vii) we get $\cos(\theta+\phi) = 0$

4 $\theta + \phi = \pi/2$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90^o.

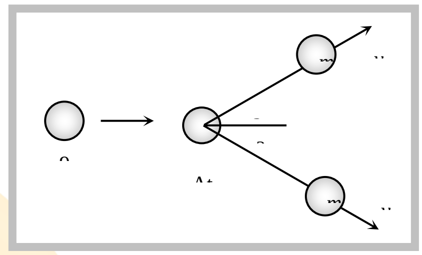
Problems based on oblique elastic collision

***Problem* 42.** A ball moving with velocity of $9 m/s$ collides with another similar stationary ball. After the collision both the balls move in directions making an angle of 30^o with the initial direction. After the collision their speed will be

(a) $2.6 m/s$ (b) $5.2 m/s$ (c) $0.52 m/s$ (d) $52 m/s$

Solution : (b) Initial horizontal momentum of the system = $m \cdot 9$

Final horizontal momentum of the system = $2mv \cos 30^o$



According to law of conservation of momentum,

$m \cdot 9 = 2mv \cos 30^o$

® $v = 5.2 \ m/s$

***Problem* 43.** A ball of mass $1kg$, moving with a velocity of $0.4 m/s$ collides with another stationary ball. After the collision, the first ball moves with a velocity of $0.3 m/s$ in a direction making an angle of 90^o with its initial direction. The momentum of second ball after collision will be (in *kg-m/s*)

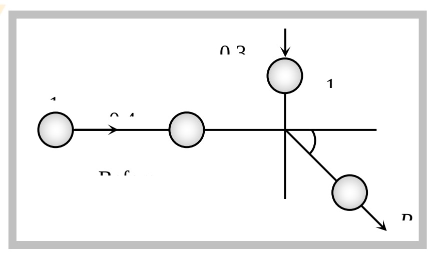
(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.7

Solution : (c) Let second ball moves with momentum *P* making an angle (from the horizontal (as shown in the figure).

By the conservation of horizontal momentum $1 \times 0.4 = P \cos \theta$ ......(i)

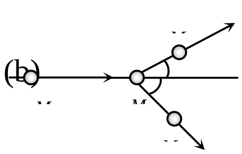
By the conservation of vertical momentum $0.3 = P \sin \theta$ ......(ii)

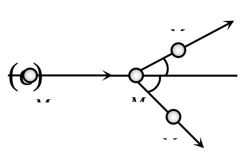
From (i) and (ii) we get $P = 0.5 \ kg\text{-}m/s$

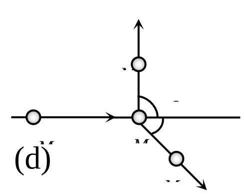


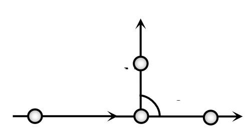
***Problem* 44.** Keeping the principle of conservation of momentum in mind which of the following collision diagram is not correct

(a) (b) (c) (d)









Solution : (d) In this condition the final resultant momentum makes some angle with *x*-axis. Which is not possible because initial momentum is along the *x*-axis and according to law of conservation of momentum initial and final momentum should be equal in magnitude and direction both.

Problem **45.** A ball B_1 of mass *M* moving northwards with velocity *v* collides elastically with another ball B_2 of same mass but moving eastwards with the same velocity *v*. Which of the following statements will be true

(a) B_1 comes to rest but B_2 moves with velocity $\sqrt{2}v$

(b) B_1 moves with velocity $\sqrt{2}v$ but B_2 comes to rest

(c) Both move with velocity $v/\sqrt{2}$ in north east direction

(d) B_1 moves eastwards and B_2 moves north wards

Solution : (d) Horizontal momentum and vertical momentum both should remain conserve before and after collision. This is possible only for the (d) option.

6.24 Head on Inelastic Collision

(1) **Velocity after collision :** Let two bodies *A* and *B* collide inelastically and coefficient of restitution is *e*.

Where $$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

® $$v_2 - v_1 = e(u_1 - u_2)$$

4 $$v_2 = v_1 + e(u_1 - u_2) \quad \text{......(i)}$$

From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{......(ii)}$$

By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

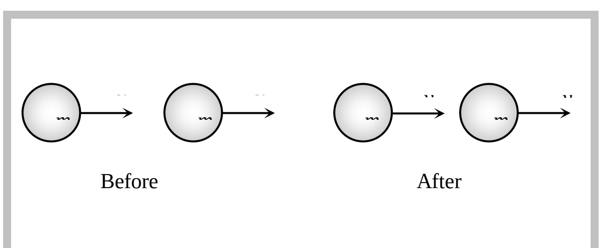
Similarly $$v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

By substituting *e* = 1, we get the value of v_1 and u_2 for perfectly elastic head on collision.

(2) **Ratio of velocities after inelastic collision :** A sphere of mass *m* moving with velocity *u* hits inelastically with another stationary sphere of same mass.

4 $$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

® $$v_2 - v_1 = eu \quad \text{......(i)}$$



By conservation of momentum :

Momentum before collision = Momentum after collision

$$mu = mv_1 + mv_2$$

® $v_1 + v_2 = u$ ……(ii)

Solving equation (i) and (ii) we get $v_1 = \frac{u}{2}(1-e)$ and $v_2 = \frac{u}{2}(1+e)$

4 $\frac{v_1}{v_2} = \frac{1-e}{1+e}$

(3) **Loss in kinetic energy**

Loss (⊗K) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Substituting the value of v_1 and v_2 from the above expression

Loss (⊗K) = $\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(1-e^2)(u_1-u_2)^2$

By substituting $e = 1$ we get ⊗$K = 0$ *i.e.* for perfectly elastic collision loss of kinetic energy will be zero or kinetic energy remains constant before and after the collision.

Problems based on inelastic collision

***Problem* 46.** One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is $\frac{1}{2}$, the ratio of their speeds after collision shall be **[RPMT 1998]**

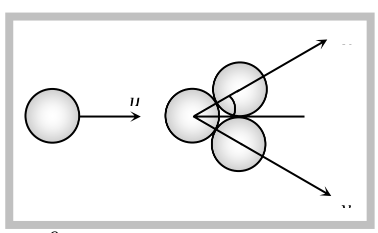
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

Solution : (c) $\frac{v_1}{v_2} = \frac{1-e}{1+e} = \frac{1-1/2}{1+1/2} = \frac{1/2}{3/2} = \frac{1}{3}$.

***Problem* 47.** Two identical billiard balls are in contact on a table. A third identical ball strikes them symmetrically and come to rest after impact. The coefficient of restitution is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{\sqrt{3}}{2}$

Solution : (a) $\sin\theta = \frac{r}{2r} = \frac{1}{2}$ ® (= 30°

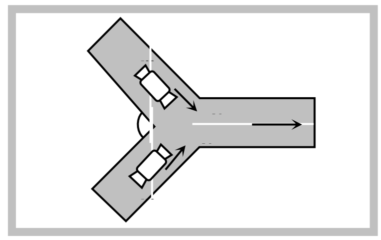


From conservation of linear momentum $mu = 2mv\cos 30^o$ or $v = \frac{u}{\sqrt{3}}$

Now $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$ in common normal direction.

Hence, $e = \frac{v}{u\cos 30^o} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$

***Problem* 48.** Two cars of same mass are moving with same speed v on two different roads inclined at an angle θ with each other, as shown in the figure. At the junction of these roads the two cars collide inelastically and move simultaneously with the same speed. The speed of these cars would be



(a) $v\cos\frac{\theta}{2}$

(b) $\frac{v}{2}\cos\theta$

(c) $\frac{v}{2}\cos\frac{\theta}{2}$

(d) $2v\cos\theta$

Solution : (a) Initial horizontal momentum of the system $= mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2}$.

If after the collision cars move with common velocity V then final horizontal momentum of the system = $2mV$.

By the law of conservation of momentum, $2mV = mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2} \Rightarrow V = v\cos\frac{\theta}{2}$.

6.25 Rebounding of Ball After Collision With Ground

If a ball is dropped from a height h on a horizontal floor, then it strikes with the floor with a speed.

$v_0 = \sqrt{2gh_0}$ [From $v^2 = u^2 + 2gh$]

and it rebounds from the floor with a speed

$v_1 = e\,v_0 = e\sqrt{2gh_0}$ $\left[\text{As } e = \frac{\text{velocity after collision}}{\text{velocity before collision}}\right]$

(1) **First height of rebound :** $h_1 = \frac{v_1^2}{2g} = e^2 h_0$

4 $h_1 = e^2 h_0$

(2) **Height of the ball after n^{th} rebound :** Obviously, the velocity of ball after n^{th} rebound will be

$v_n = e^n v_0$

Therefore the height after n^{th} rebound will be $h_n = \frac{v_n^2}{2g} = e^{2n} h_0$

4 $h_n = e^{2n} h_0$

(3) **Total distance travelled by the ball before it stops bouncing**

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + ... = h_0 + 2e^2 h_0 + 2e^4 h_0 + 2e^6 h_0 + ...$$

$$H = h_0[1 + 2e^2(1 + e^2 + e^4 + e^6 ....)] = h_0\left[1 + 2e^2\left(\frac{1}{1-e^2}\right)\right] \qquad \left[\text{As } 1 + e^2 + e^4 + ... = \frac{1}{1-e^2}\right]$$

4 $$H = h_0\left[\frac{1+e^2}{1-e^2}\right]$$

(4) **Total time taken by the ball to stop bouncing**

$$T = t_0 + 2t_1 + 2t_2 + 2t_3 + ... = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$$

$$= \sqrt{\frac{2h_0}{g}}\ [1 + 2e + 2e^2 + ...] \qquad [\text{As } h_1 = e^2 h_0;\ h_2 = e^4 h_0]$$

$$= \sqrt{\frac{2h_0}{g}}\ [1 + 2e(1 + e + e^2 + e^3 + ...)] = \sqrt{\frac{2h_0}{g}}\left[1 + 2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

4 $$T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h_0}{g}}$$

Sample problems based on rebound of ball after collision with ground

Problem **49.** A rubber ball is dropped from a height of $5m$ on a planet where the acceleration due to gravity is not known. On bouncing, it rises to $1.8m$. The ball loses its velocity on bouncing by a factor of **[CBSE PMT 1998]**

(a) 16/25 (b) 2/5 (c) 3/5 (d) 9/25

Solution : (c) If ball falls from height h_1, then it collides with ground with speed $v_1 = \sqrt{2gh_1}$ …..(i)

and if it rebound with velocity v_2, then it goes upto height h_2 from ground, $v_2 = \sqrt{2gh_2}$ …..(ii)

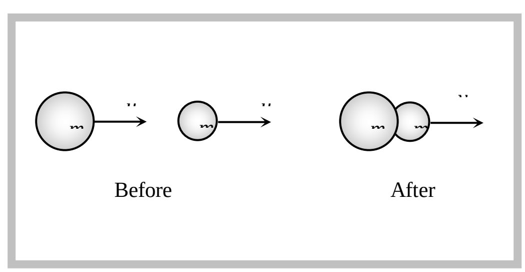
From (i) and (ii) $\frac{v_2}{v_1} = \sqrt{\frac{2gh_2}{2gh_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

6.26 Perfectly Inelastic Collision

In such types of collisions the bodies move independently before collision but after collision as a one single body.

(1) **When the colliding bodies are moving in the same direction**

By the law of conservation of momentum



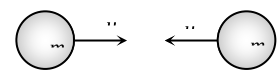
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v_{\text{comb}}$$

® $$v_{\text{comb}} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Loss in kinetic energy $\Delta K = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v_{comb}^2$

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$ [By substituting the value of v_{comb}]

(2) **When the colliding bodies are moving in the opposite direction**



Before

By the law of conservation of momentum

$m_1 u_1 + m_2(-u_2) = (m_1 + m_2)v_{\text{comb}}$ (Taking left to right as positive)

4 $v_{\text{comb}} = \frac{m_1 u_1 - m_2 u_2}{m_1 + m_2}$

when $m_1 u_1 > m_2 u_2$ then $v_{\text{comb}} > 0$ (positive)

i.e. the combined body will move along the direction of motion of mass m_1.

when $m_1 u_1 < m_2 u_2$ then $v_{\text{comb}} < 0$ (negative)

i.e. the combined body will move in a direction opposite to the motion of mass m_1.

(3) **Loss in kinetic energy**

⊗K = Initial kinetic energy – Final kinetic energy

$$= \left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)v_{\text{comb}}^2\right)$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2$$

Problems based on perfectly inelastic collision

***Problem* 49.** Which of the following is not a perfectly inelastic collision **[BHU 1998; JIPMER 2001, 2002]**

(a) Striking of two glass balls

(b) A bullet striking a bag of sand

(c) An electron captured by a proton

(d) A man jumping onto a moving cart

Solution : (a) For perfectly elastic collision relative velocity of separation should be zero *i.e.* the colliding body should move together with common velocity.

***Problem* 50.** A metal ball of mass $2kg$ moving with a velocity of $36 km/h$ has an head-on collision with a stationary ball of mass $3kg$. If after the collision, the two balls move together, the loss in kinetic energy due to collision is

(a) $40\,J$

(b) $60\,J$

(c) $100\,J$

(d) $140\,J$

Solution : (b) Loss in kinetic energy $\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2 = \frac{1}{2}\frac{2 \times 3}{2 + 3}(10 - 0)^2 = 60\,J$.

***Problem* 51.** A ball moving with speed v hits another identical ball at rest. The two balls stick together after collision. If specific heat of the material of the balls is S, the temperature rise resulting from the collision is

(a) $\frac{v^2}{8S}$

(b) $\frac{v^2}{4S}$

(c) $\frac{v^2}{2S}$

(d) $\frac{v^2}{S}$

Solution : (b) Kinetic energy of ball will raise the temperature of the system $\frac{1}{2}mv^2 = (2m)S\,\Delta t$ ® $\Delta t = \frac{v^2}{4S}$.

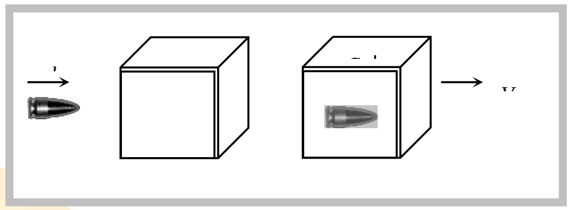
Problem **52.** A bullet of mass *a* is fired with velocity *b* in a large block of mass *c*. The final velocity of the system will be

(a) $\frac{c}{a+c}$ (b) $\frac{ab}{a+c}$ (c) $\frac{(a+b)}{c}$ (d) $\frac{(a+c)}{a}b$

Solution : (b) Initially bullet moves with velocity *b* and after collision bullet get embedded in block and both move together with common velocity.

By the conservation of momentum $a \cdot b + 0 = (a + c)\,V$

4 $V = \frac{ab}{a+c}$

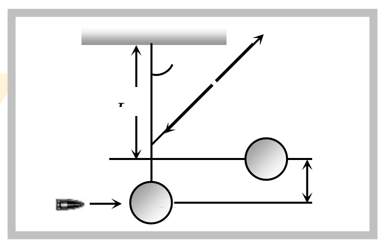


6.27 Collision Between Bullet and Vertically Suspended Block

A bullet of mass *m* is fired horizontally with velocity *u* in block of mass *M* suspended by vertical thread. After the collision bullet gets embedded in block. Let the combined system raised upto height *h* and the string makes an angle (with the vertical.

(1) **Velocity of system**

Let *v* be the velocity of the system (block + bullet) just after the collision.



$\text{Momentum}_{\text{bullet}} + \text{Momentum}_{\text{block}} = \text{Momentum}_{\text{bullet and block system}}$

$$mu + 0 = (m + M)v$$

4 $$v = \frac{mu}{(m+M)} \quad \ldots\ldots(i)$$

(2) **Velocity of bullet :** Due to energy which remains in the bullet block system, just after the collision, the system (bullet + block) rises upto height *h*.

By the conservation of mechanical energy $\frac{1}{2}(m+M)v^2 = (m+M)gh$ ® $v = \sqrt{2gh}$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \frac{mu}{m+M}$

4 $$u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy :** We know the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2$$

4 $$\Delta K = \frac{1}{2}\frac{mM}{m+M}u^2$$ [As $u_1 = u$, $u_2 = 0$, $m_1 = m$ and $m_2 = M$]

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \frac{u^2}{2g}\left(\frac{m}{m+M}\right)^2$

From the figure $\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{u^2}{2gL}\left(\frac{m}{m+M}\right)^2$

or

$$\theta = \cos^{-1}\left[1 - \frac{1}{2gL}\left(\frac{mu}{m+M}\right)^2\right]$$

Problems based on collision between bullet and block

***Problem* 53.** A bullet of mass m moving with velocity v strikes a block of mass M at rest and gets embeded into it. The kinetic energy of the composite block will be **[MP PET 2002]**

(a) $\frac{1}{2}mv^2 \times \frac{m}{(m+M)}$ (b) $\frac{1}{2}mv^2 \times \frac{M}{(m+M)}$ (c) $\frac{1}{2}mv^2 \times \frac{(M+m)}{M}$ (d) $\frac{1}{2}Mv^2 \times \frac{m}{(m+M)}$

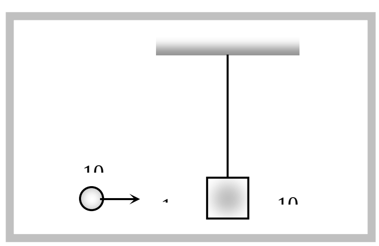
Solution : (a) By conservation of momentum,

Momentum of the bullet (mv) = momentum of the composite block $(m+M)V$

® Velocity of composite block $V = \frac{mv}{m+M}$

4 Kinetic energy $= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{m+M} = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$.

***Problem* 54.** A mass of $10\,gm$, moving horizontally with a velocity of $100\,cm/\sec$, strikes the bob of a pendulum and strikes to it. The mass of the bob is also $10\,gm$ (see fig.) The maximum height to which the system can be raised is ($g = 10\,m/\sec^2$)



(a) Zero

(b) $5\,cm$

(c) $2.5\,cm$

(d) $1.25\,cm$

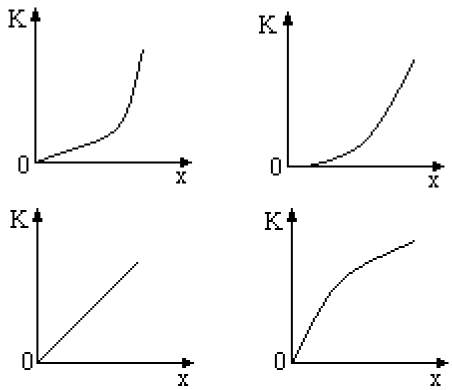
Solution : (d) By the conservation of momentum,

Momentum of the bullet = Momentum of system ® $10 \times 1 = (10 + 10) \times v$ ® $v = \frac{1}{2}\,m/s$

Now maximum height reached by system $H_{max} = \frac{v^2}{2g} = \frac{(1/2)^2}{2 \times 10} m = 1.25 cm$.

EXAMPLE – I

1. A body moves from rest with a constant acceleration. Which one of the following graphs represents the variation of its kinetic energy K with the distance traveled x?



2. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in a time t will be

(a) zero (b) mgvt $\cos^2\theta$ (c) mgvt $\sin^2\theta$ (d) $\frac{1}{2} mgvt \sin 2\theta$

3. The total work done on a particle is equal to the change in its kinetic energy

(a) always

(b) only if the conservative forces are acting on it.

(c) only in an inertial frame.

(d) only if non-conservative forces are absent.

4. A particle is acted upon by a conservative force F = $(7\hat{i} - 6\hat{j})N$. The work done by the force when the particle moves form origin (0,0) position (-3m, 4m) is given by

(a) 3J (b) 10J (c) -45J (d) none of these

5. No work is done by a force on a particle if

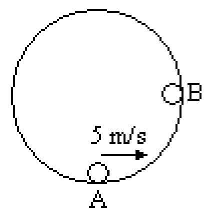
(a) the particle remains stationary

(b) the force is always perpendicular to its velocity

(c) the force is always parallel to its velocity

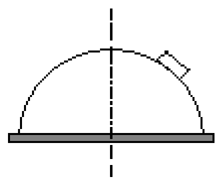
(d) the velocity of particle is unaffected by the application.

6. A ball of mass 1kg moves inside a smooth spherical shell of radius 1m with an initial velocity v = 5m/s from the bottom. What is the total force acting on the particle at point B



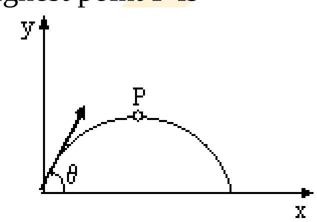
(a) 10N (b) 25N (c) 5□5N (d) 5N

7. A particle of mass m slides down a smooth hemisphere of radius R. The height at which the body be breaks off from the hemisphere is



(a) h = R/2 (b) h = R/3 (c) h = 2R/3 (d) h = R/4

8. A particle is projected with an initial velocity u at angle θ from the ground. What is the work done by gravity during the time it reaches the highest point P is



(a) $\frac{-mu^2 \sin^2\theta}{2}$ (b) $+\frac{mu^2 \sin^2\theta}{2}$ (c) zero (d) +mu sinθ

9. A particle of mass m is located in a one dimensional potential field where potential energy of the particle has the form $U(x) = \frac{a}{x^2} - \frac{b}{x}$

(a) $\frac{b}{2a}$ (b) $\frac{2b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{2a}{b}$

10. A man pulls a bucket of water from a well of depth H. If the mass of the rope and that of the bucket full of water are m and M respectively, then the work done by the man is

(a) (m + M)gh (b) $\left(\frac{m}{2} + M\right)gh$ (c) $\left(\frac{m+M}{2}\right)gh$ (d) $\left(m + \frac{M}{2}\right)gh$

11. The kinetic energy of a particle moving along a straight line increases uniformly with respect to the distance traveled by it. The force on the particle is

(a) constant
(b) proportional to velocity
(c) inversely proportional to velocity
(d) directly proportional to the square root of velocity.

12. The potential difference applied to an X – ray tube is V. The ratio of debroglie wavelength of X-ray is directly proportional to

(a) V (b) $V^{1/2}$ (c) $\frac{1}{\sqrt{V}}$ (d) independent of V

13. A particle of mass 0.75kg on a horizontal plane under action of a single force F = $\left(3\hat{i}+3\hat{j}\right)$ N. Under this force it is displaced form (0,0)) to (1m, 1m). The initial speed of the particle is 3m/s, its final speed will be

(a) $4ms^{-1}$ (b) $\sqrt{20}ms^{-1}$ (c) 5 ms-1 (d) $\sqrt{12}ms^{-1}$

Exercise I

1. c	2. c	3. a	4. c	5. a, b
6. c	7. c	8. a	9. d	10. b
11. a	12.	13. c		