

**GRAVITATION: PLANET & SATALIGHT****Syllabus:- Gravitation**

*Kepler's laws of planetary motion. The universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth.*

*Gravitational potential energy; gravitational potential. Escape velocity, orbital velocity of a satellite. Geostationary satellites.*

**KEPLER'S LAWS**

Kepler discovered his three laws of planetary motion by analyzing the observational tables prepared by Tycho Brahe in the sixteenth century showing the angular position of planets at different instants. These three laws are stated as:

**Law of Orbits**

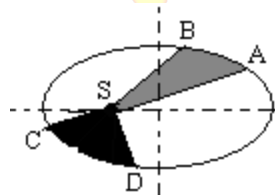
The planets move around the sun in elliptical orbits with the sun at one focus.

**Law of Areas**

The line joining the sun to a planet sweeps out equal areas in equal time.

The area SAB and SCD are equal since they are swept out by the radial line in equal interval of time.

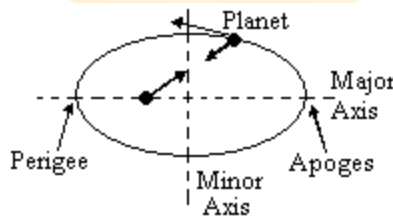
This law is a consequence of **conservation angular momentum**.

**Laws of Periods**

The square of the time period of any planet is proportional to the cube of the semi-major axis of its orbit, i.e.

$$T^2 \propto a^3$$

$$T^2 \propto ka^3$$



Where k is a constant applicable to the planets.

Note that the first two laws speak about the orbit of the planet while the third law relates the orbit of one planet with that of the other.

**Newton's Law of Gravitation**

**It states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.**

Consider two bodies A and B of masses  $m_1$  and  $m_2$ . Let  $R$  be the distance between their centre and  $F$  be the force of attraction between them,

According to Newton's law of Gravitation,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\text{or } F = \frac{Gm_1 m_2}{r^2} \dots\dots\dots(1)$$

where  $G$  is a constant of proportionality and is called Universal Gravitational Constant.

**Definition of  $G$ .**

Let  $m_1 = m_2 = 1$  and  $r = 1$

$$F = G \frac{1 \times 1}{1^2} = G$$

Then from (1),

$$\text{or } G = F$$

Thus Universal Gravitational Constant is equal to the force of attraction acting between two bodies each of unit mass, whose centres are placed unit distance apart.

**Gravitational constant is a scalar quantity. Its value is same throughout the universe and is independent of the nature and the size of the bodies as well as the nature of the medium between the bodies. The value of  $G$  in S.I. is  $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$  and in cgs system is  $6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$ .**

**Dimensional formula for  $G$**

$$G = \frac{Fr^2}{m_1 m_2} = \frac{(MLT^{-2})(L^2)}{M \times M}$$

$$= [M^{-1} L^3 T^{-2}]$$

**Relation between  $g$  and  $G$**

Consider earth to be a spherical body has uniform density and its mass  $M$  can be supposed to be concentrated at its centre  $O$  of radius  $R$ . Suppose a body of mass  $m$  placed on the surface of earth, where acceleration due to gravity is  $g$ .

Let  $F$  be the force of attraction between body and the earth.

According to Newton's law of gravitation

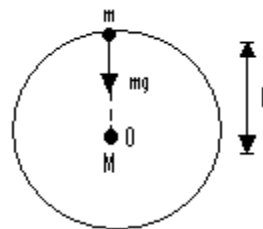
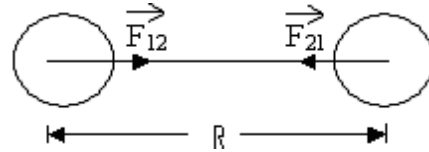
$$F = \frac{GMm}{R^2} \dots\dots\dots(2)$$

due to gravity,  $F = mg \dots\dots\dots(3)$

from (i) & (ii)

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} \dots\dots\dots(4)$$



Thus value of acceleration due to gravity is **independent** of **mass, shape and size** of the body but **depends** upon **mass and radius** of the earth.

### Variation of Acceleration due to Gravity

The value of acceleration due to gravity changes with height (i.e. altitude), depth, shape of the earth and rotation of earth about its own axis.

**Effect of height (Altitude).** Consider earth to be a sphere of mass  $M$ , radius  $R$  with centre at  $O$ . Let  $g$  be the value of acceleration due to gravity at a point  $A$  on the surface of earth,

$$\therefore g = \frac{GM}{R^2}$$

If  $g'$  is the acceleration due to gravity at a point, at a height  $h$  above the surface of earth, then

$$g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$$

$$\boxed{g' = g \left( \frac{R^2}{(R+h)^2} \right)} \dots\dots\dots(5)$$

$$\frac{g'}{g} = \frac{1}{\frac{(R+h)^2}{R^2}}$$

$$= \frac{1}{\left(1 + \frac{h}{R}\right)^2} = \left(1 + \frac{h}{R}\right)^{-2} = \left(1 - \frac{2h}{R}\right)$$

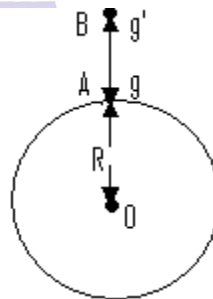
If  $h \ll R$  then  $h/R$  is very small as compared to 1

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$\text{or } g' = g \left(1 - \frac{2h}{R}\right) \dots\dots\dots(6)$$

, We note that the value of acceleration due to gravity decreases with height.

It is due to this reason that the value of acceleration due to gravity is lesser at mountains than in plane. At a height equal to the radius of the earth (i.e.  $h = R = 6400 \text{ km}$ ),



$$g' = \frac{gR^2}{(R+h)^2} = \frac{g}{4}$$

Here, we have not apply eqn. (6) since  $h \ll R$  is not valid.

### Important notes

- (i) The relation (5) is used to find the value of acceleration due to gravity at a height when  $h$  is comparable to the radius of earth  $R$  and relation (6) is used to find  $g'$  when  $h$  is very small as compared to  $R$ .

- (ii) With height  $h$ , the decrease in the value of  $g$  is  $g - g' = 2hg/R$

$\therefore$  Fractional decrease in the value of  $g$

$$= \frac{g - g'}{g} = \frac{2h}{R}$$

$\therefore$  % decrease in the value of  $g$

$$= \left( \frac{g - g'}{g} \right) \times 100 = \frac{2h}{R} \times 100$$

**Effect of depth.** Consider earth to be a homogeneous sphere of radius  $R$  and mass  $M$  with centre at  $O$ . Let  $g$  be the value of acceleration due to gravity at a point  $A$  on the surface of earth. Then

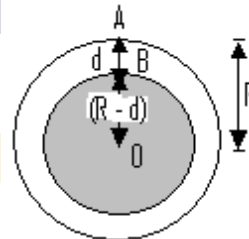
$$g = \frac{GM}{R^2}$$

If  $\rho$  is uniform density of material of the earth,

$$M = \frac{4}{3} \pi R^3 \rho$$

then

$$\therefore g = \frac{G \times \frac{4}{3} \pi R^3 \rho}{R^2} = \frac{4}{3} \pi G R \rho$$



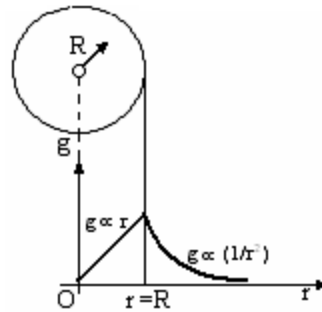
Let  $g'$  be the acceleration due to gravity at the point  $B$  at a depth  $d$  below the surface of earth. The body at  $B$  will experience gravity pull due to shaded portion of earth whose radius is  $(R - d)$  and mass is  $M'$

$$\therefore g' = \frac{GM'}{(R-d)^2}$$

$$\text{and } M' = \frac{4}{3}\pi(R-d)^3 \rho$$

$$\therefore g' = \frac{G \times \frac{4}{3}\pi(R-d)^3 \rho}{(R-d)^2}$$

$$= \frac{4}{3}\pi G(R-d)\rho$$



$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi GR\rho}$$

$$= \frac{R-d}{R} = \frac{R}{R} - \frac{d}{R}$$

$$g' = g \left( 1 - \frac{d}{R} \right) \dots\dots\dots(7)$$

From eqn (7) the value of acceleration due to gravity decreases with depth.

#### Important notes

##### At centre of the earth

$d = R$ , we get

$g' = 0$ .

The weight of body of mass  $m$  at centre of the earth is zero.

The decrease in the value of  $g$  with depth  $d$  is  $= g - g' = dg/R$

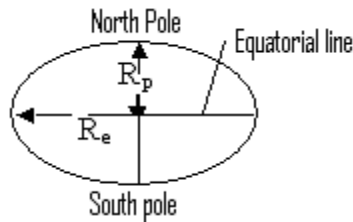
$\therefore$  Fractional decrease in the value of  $g$

$$= \frac{g - g'}{g} = \frac{d}{R}$$

$\therefore$  % decrease in the value of  $g$

$$= \left( \frac{g - g'}{g} \right) \times 100 = \frac{d}{R} \times 100$$

**Effect of Shape of Earth.** Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. Equatorial radius  $R_e$  of the earth is about 21 km greater than the polar radius  $R_p$ ,



Now,  $g = GM/R^2$

Since  $G$  and  $M$  are constants

$$g \propto 1/R^2$$

Thus we conclude that the value of  $g$  is least at the equator and maximum at the pole. It means, the value of acceleration due to gravity increases as we go from equator to the pole.

At sea level, the value of  $g$  at pole is greater than its value at equator by  $1.80 \text{ cms}^{-2}$ .

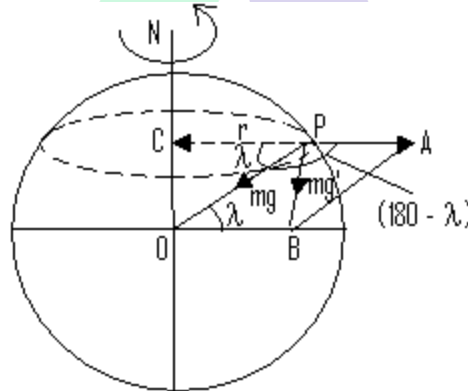
#### Effect of latitude (Due to Rotation of earth about its own axis)

**Latitude at a place** is defined as the angle, which the line joining the place to the centre of the earth makes with the equatorial plane. At a place  $P$  latitude angle  $= \angle POB = \lambda$ ,  $\lambda = 90^\circ$  at poles and  $\lambda = 0^\circ$  at equator.

Consider earth to be a perfect sphere of mass  $M$ , radius  $R$  with centre  $O$ . Consider a particle of mass  $m$  at a place  $P$  of latitude  $\lambda$ . If earth is rotating about its polar axis  $NS$  with constant angular velocity  $\omega$ , then particle at  $P$  rotates of radius  $r$ .

$$r = PC = OP \cos \lambda = R \cos \lambda$$

The centrifugal force acting on the particle at  $P$  is  $mr\omega^2$ . it acts along  $PA$ ,



Suppose  $g$  is the acceleration due to gravity when earth were at rest. Let  $g'$  be the acceleration due to gravity at  $P$  when rotation of earth is taken into account.

Then apparent weight of the particle at  $P = mg'$

This is the resultant of the true weight  $mg$  and centrifugal force  $mr\omega^2$  acting at  $P$  and hence must be represented by the diagonal  $PB$ . Here  $\angle APO = (180^\circ - \lambda)$ .

Using parallelogram law of forces, we have

$$mg' = \sqrt{(mg)^2 + (mr\omega^2)^2 + 2(mg)(mr\omega^2)\cos(180^\circ - \lambda)}$$

$$\text{or } g' = \sqrt{g^2 + r^2\omega^4 - 2gr\omega^2\cos\lambda}$$

$$= \sqrt{g^2 + R^2\cos^2\lambda\omega^4 - 2gR\cos\lambda\omega^2\cos\lambda}$$

$$= g \left( 1 + \frac{R^2 \omega^4}{g^2} \cos^2 \lambda - \frac{2R\omega^2}{g} \cos^2 \lambda \right)^{1/2}$$

We know that,  $R = 6.4 \times 10^6 \text{ m}$ ,  
 $g = 9.8 \text{ ms}^{-2}$

and  $\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} \text{ rads}^{-1}$ .

$$\frac{R\omega^2}{g} = 6.4 \times 10^6 \times \left( \frac{2\pi}{24 \times 60 \times 60} \right)^2 \times \frac{1}{9.8}$$

$$= \frac{1}{289}$$

Since the value of  $R\omega^2/g$  is very small, therefore the terms with its squares and of higher powers can be neglected.

Hence, becomes

$$g' = g \left( 1 - \frac{2R\omega^2 \cos^2 \lambda}{g} \right)^{1/2}$$

Expanding it by Binomial theorem, we get

$$g' = g \left( 1 - \frac{1}{2} \times \frac{2R\omega^2 \cos^2 \lambda}{g} \right)$$

$$= g \left( 1 - \frac{R\omega^2 \cos^2 \lambda}{g} \right)$$

$$g' = g - R\omega^2 \cos^2 \lambda \dots\dots\dots(8)$$

As,  $\cos \lambda$  and  $\omega$  are positive, therefore  $g' < g$ . Thus from, it is clear that acceleration due to gravity

- (i) decreases on account of rotation of earth,
- (ii) increase with the increase in altitude of the place.

### HOW DOES ONE PARTICLE EXERTS FORCE ON THE OTHER ?

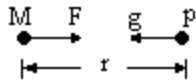
The interaction between the two point masses is visualized in terms of the concept of gravitational field. Every mass particle is surrounded by a space within which its influence can be felt. This region or space is said to be occupied with gravitational field. Every point in the region of gravitational field is associated with two characteristics of field:

- gravitational field strength or intensity  $g$  ( a vector)
- gravitational potential  $V$  ( a scalar)

### GRAVITATIONAL FIELD STRENGTH ( $g$ )

At a point in gravitational field, it is defined as the force experienced by a unit mass placed at that point. Gravitational field strength at point  $p$  due a point mass  $M$ , which at a distance  $r$  is given by

$$\mathbf{g} = \frac{-GM}{r^2} \hat{r}$$



The negative sign shows that it is always directed towards the source particle. If a point mass  $m$  is placed at  $p$ , it experience a force given by

$$\mathbf{F} = m\mathbf{g}$$

### An uniform solid Sphere of Mass $M$

Outside: To any point lying outside the sphere, It behaves like a point mass.

$$g = \frac{GM}{r^2} \quad r \geq R$$

Inside: To a point at a distance  $r < R$  from the centre, the portion of the sphere that lies outside the radius  $r$  does not contribute to the field.

Therefore,

$$g = \frac{GM'}{r^2} \quad r < R$$

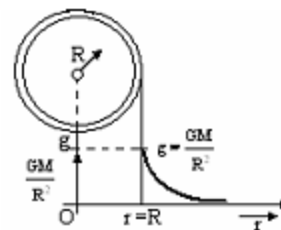
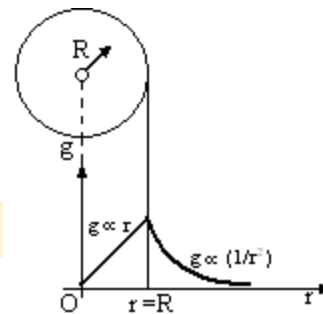
Inside:

### A Spherical Shell of Mass $M$ .

**Outside:** It behave likes point mass.

$$\text{So, } g = \frac{GM}{r^2} \quad r \geq R$$

**Inside:** A point mass placed anywhere inside the shell is equally attracted in all directions. So  $g = 0$  ;  $r < R$



## GRAVITATIONAL POTENTIAL (V)

The gravitational potential at a point in gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

Gravitational potential at a point is defined as the potential energy per unit mass at that point.

The potential energy at a point is always defined with respect to a reference point, which is usually at infinity, we know that negative of the work done by conservative force gives the change in **potential energy** between two points,

$$U_B - U_A = - \int_A^B \mathbf{F}_c \cdot d\mathbf{s}$$

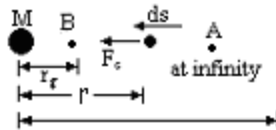


Let us assume that the point A is at infinity, thus  $U_A = 0$

For two point masses M and m separated at a distance r, the force of interaction is given by

$$F_c = \frac{GMm}{r^2}$$

thus,



$$F_c \cdot ds = F_c dr = \frac{GMm}{r^2} dr = -\frac{GMm}{r^2} (-dr) \quad (\text{ } ds = -dr)$$

$$\int F \cdot ds = -GMm \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

$$= -GMm \left[ -\frac{1}{r} \right]_{r_A}^{r_B}$$

$$= -GMm \left( -\frac{1}{r_B} + \frac{1}{r_A} \right)$$

$$\therefore U_B - U_A = GMm \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

Since for  $r_A = \infty$ ;  $U_A = 0$ , therefore.

$$U_B = \frac{-GMm}{r_B}$$

In general, the potential energy at a distance r from a point mass M is

$$U = \frac{-GMm}{r} \dots\dots\dots(9)$$

Note that the negative sign shows that the gravitational force is attractive. Hence the gravitational potential at a distance r from a point mass M is

$$V = \frac{U}{m} = \frac{-GM}{r}$$

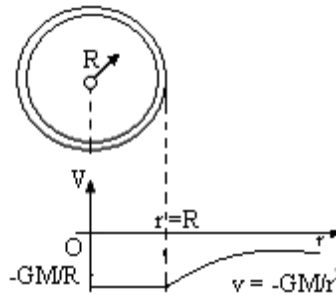
### (i) A spherical Shell of Mass M

Outside: It behaves like a point mass. Therefore,

$$V = \frac{-GM}{r} \quad r \geq R$$

Inside: since no gravitational field exists inside the shell, therefore, whatever is the gravitational potential at the surface the same continues inside it also. Thus,

$$V = \frac{-GM}{r} \quad r \geq R$$



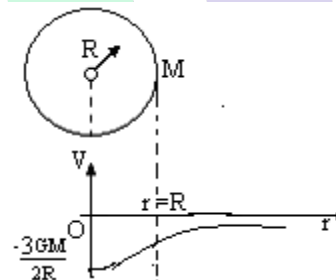
### (ii) An uniform Solid Sphere of Mass M

Outside: It behaves like a point mass.

$$V = \frac{-GM}{r} \quad r \geq R$$

$$V = \frac{-GM}{r} \left[ 3 - \left( \frac{r}{R} \right)^2 \right] \quad r \geq R$$

Inside:



### Orbital Velocity, Time Period and Height of Satellite

**Orbital velocity.** Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth. The value of orbital velocity is different for different orbits around earth and is independent of the mass of satellite. Let

M = mass of earth,

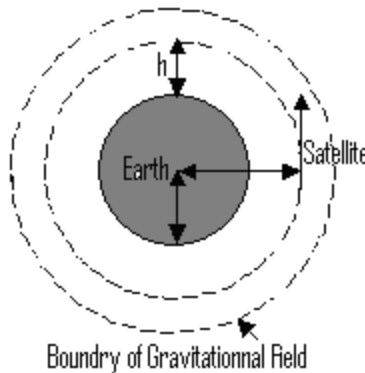
R = radius of earth,

m = mass of the satellite,

v = orbital velocity of the satellite,

h = height of satellite above the surface of earth,

$r$  = radius of the satellite  
 $= R + h$



The centripetal force required to keep the satellite in its orbit,

$$F = mv^2/r$$

According to Newton's Law of Gravitation,

$$\therefore \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\text{or } v = \sqrt{\frac{GM}{r}} \dots\dots\dots(10)$$

If  $g$  is the value of acceleration due to gravity on the surface of earth, then

$$g = GM/R^2 \text{ or } GM = gR^2$$

Putting this value in equation (10), we get

$$v = \sqrt{\frac{gR^2}{r}} = R\sqrt{\frac{g}{R+h}} \dots\dots\dots(11)$$

orbital velocity of a satellite

- (i) is independent of the mass of satellite.
- (ii) decreases with an increase in the radius of orbit or increase in the height of satellite.
- (iii) depends upon the mass and radius of the earth/planet around which the revolution of satellite is taking place.

The direction of orbit velocity of a satellite at an instant is along the tangent to the orbital path of satellite at that instant.

When a satellite is orbiting very close to the surface of earth  $h \ll R$ , then

$$r = R + h = R$$

$$v = v_o$$

$$v_o = R \sqrt{g/R} = \sqrt{gR}$$

$$g = 9.8 \text{ ms}^{-2}$$

$$R = 6.4 \times 10^6 \text{ m.}$$

$$v_o = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.92 \times 10^3 \text{ ms}^{-1}$$

$$= 7.92 \text{ kms}^{-1}$$

**Time period of satellite.** It is the time taken by satellite to complete one revolution

around the earth and is denoted by T. Thus

$$T = \frac{\text{distance travelled in one revolution}}{\text{orbital velocity}}$$

$$= \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{R} \sqrt{\frac{r}{g}} = \frac{2\pi}{R} \sqrt{\frac{r^3}{g}}$$

$$= \frac{2\pi}{R} \sqrt{\frac{(R+h)^3}{g}} \dots\dots\dots(12)$$

If the earth is supposed to be a sphere of mean density  $\rho$ , then the mass of the earth is

$$M = \frac{4}{3}\pi R^3 \rho \quad \text{and}$$

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \left( \frac{4}{3}\pi R^3 \rho \right) = \frac{4\pi GR\rho}{3}$$

Substituting this value of g in eqn (12), we get

$$T = \frac{2\pi}{R} \sqrt{\frac{3(R+h)^3}{4\pi GR\rho}}$$

$$T = \sqrt{\frac{4\pi^2}{R^2} \times \frac{3(R+h)^3}{4\pi GR\rho}}$$

$$= \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}} \dots\dots\dots(13)$$

For satellite orbiting close to the surface of earth,  $h \ll R$

$\therefore h + R = R$ . hence, from eqn (13)

$$T = \sqrt{\frac{3\pi}{G\rho}} \dots\dots\dots(14)$$

From eqn(12)

$$T = \frac{2\pi}{R} \sqrt{\frac{R^3}{g}}$$

$$T = 2\pi \sqrt{\frac{R}{g}} \dots\dots\dots(15)$$

By substituting  $g = 9.8 \text{ ms}^{-2}$  and  $R = 6.4 \times 10^6 \text{ m}$  we get the value of  $T = 5.08 \times 10^3 \text{ sec.} = 84.6 \text{ minutes}$

*It means the satellite orbiting close to the surface of earth has a time period of revolution about 84.6 minutes.*

It is clear from eqn(13) period of revolution of a satellite depends only upon its height above the earth's surface. The larger is the height of a satellite above the surface of earth, greater will be its period of revolution.

**Altitude or Height of satellite above the earth's surface.**

Squaring both sides eqn (12) we get

$$T^2 = \frac{4\pi^2(R+h)^3}{R^2 g}$$

$$(R+h)^3 = \frac{T^2 R^2 g}{4\pi^2}$$

$$R+h = \left( \frac{T^2 R^2 g}{4\pi^2} \right)^{1/3}$$

$$h = \left( \frac{T^2 R^2 g}{4\pi^2} \right)^{1/3} - R \quad \dots\dots\dots(16)$$

Knowing the values of T, R and g, the height h of the satellite above the surface of earth can be calculated.

**Angular Momentum (L).** When a satellite of mass m is orbiting with linear speed v on the orbit of radius r around the earth, its angular momentum is given by

$$L = mvr = mr\sqrt{GM/r} = [m^2 GNr]^{1/2} \quad [\text{From eqn .10}]$$

From above, it is clear that angular momentum of a satellite depends on both, the mass of the satellite (m) and the mass of planet (M). It also depends upon the radius of the orbit of the satellite.

**Energy of Satellite.** The total mechanical energy of a satellite revolving around the earth is the sum of its potential energy (U) and kinetic energy (K).

The potential energy of a satellite is due to its position w.r.t earth. It appears because of gravitational pull acting on satellite due to earth. If a satellite of mass m is revolving around the earth of mass M, radius R with orbital velocity v, in an orbit of radius r, then potential energy of satellite is

$$U = -GMm/r$$

The kinetic energy of a satellite,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) \quad \boxed{v = \sqrt{GM/r}}$$

The total mechanical energy of a satellite is,

$$E = U + K$$

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r} = -\frac{GMm}{2r}$$

$$\text{or } E = -\frac{GMm}{2r} \quad \dots\dots\dots(17)$$

If the satellite is orbiting close to earth, then  $r = R$ . Now, total energy of satellite,

$$E = -\frac{GMm}{2R} \dots\dots\dots(18)$$

**Binding energy of a satellite.** The energy required to remove the satellite from its orbit around the earth to infinity is called **Binding energy** of the satellite. Binding energy is equal to negative value of total mechanical energy of a satellite in its orbit.

Thus binding energy = -  $E = \frac{GMm}{2r}$

### Escape Velocity

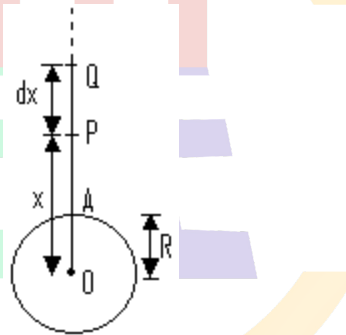
*Escape velocity on earth (or other planet) is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own.*

Let earth be a perfect sphere of mass  $M$ , radius  $R$  with centre at  $O$ . Let a body of mass  $m$  to be projected from a point  $A$  on the surface of earth.. Take two points  $P$  and  $Q$  at a distance  $x$  and  $(x + dx)$  from the centre  $O$  of the earth.

Gravitational force at  $P$  is  $F = \frac{GMm}{x^2}$

Work done in taking the body against gravitational attraction from  $P$  to  $Q$  is

$$dW = Fdx = \frac{GMm}{x^2} dx.$$



Total work done in taking the body against gravitational attraction from surface of earth (i.e.  $x = R$ ) to a region beyond the gravitational field of earth (i.e.  $x = \infty$ ) can be calculated by integrating the above expression within the limits  $x = R$  to  $x = \infty$ . Thus total work done is

$$\begin{aligned} W &= \int_R^{\infty} \frac{GMm}{x^2} dx = GMm \int_R^{\infty} x^{-2} dx \\ &= GMm \left[ \frac{x^{-2+1}}{-2+1} \right]_R^{\infty} = -GMm \left[ \frac{1}{x} \right]_R^{\infty} \\ &= -GMm \left[ \frac{1}{\infty} - \frac{1}{R} \right] = \frac{GMm}{R} \end{aligned}$$

Thus work done is at the cost of kinetic energy given to the body at the surface of the earth. If  $v_e$  is the escape velocity of the body projected from the surface of earth, then

$$\text{Kinetic energy of the body} = \frac{1}{2}mv_e^2$$

$$\therefore \frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$v_e^2 = 2GM / R$$

$$v_e = \sqrt{2GM / R} \dots\dots\dots(19)$$

$$g = GM / R^2$$

$$GM = gR^2$$

$$v_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} \dots\dots\dots(20)$$

If  $\rho$  is the mean density of the material of earth then,

$$M = \frac{4}{3}\pi R^3 \rho$$

$$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho}$$

$$= \sqrt{\frac{8\pi\rho GR^2}{3}}$$

Equation give different expressions for the escape velocity of the body. It is  $11.2 \text{ Kms}^{-1}$  of earth surface. The escape velocity of Sun is  $618 \text{ Kms}^{-1}$

### Geostationary or Geosynchronous Satellites

*A satellite which appears stationary to an observer on earth is called geostationary satellite.*

This satellite is also called as geosynchronous satellite as its angular speed is synchronized with the angular speed of the earth about its axis i.e. this satellite revolves around the earth with the same angular speed in the same direction as is done by earth around its axis.

Clearly,  $T = 24$  hours for a geostationary satellite.

To calculate the height of geostationary satellite, we use the relation as

$$h = \left( \frac{T^2 R^2 g}{4\pi^2} \right) - R$$

Putting  $R = 6.4 \times 10^6 \text{ m}$ ,  $g = 9.8 \text{ ms}^{-2}$ ,  $T = 24 \text{ hours} = 24 \times 60 \times 60 \text{ s}$ ,  
we get,  **$h = 3.6 \times 10^7 \text{ m} = 36000 \text{ km}$** .

The orbital velocity of such a satellite, using relation is found to be  $3.1 \text{ kms}^{-1}$ .

Geostationary satellites are used for the purposes of communication as they act as reflectors of suitable waves carrying the messages. Therefore, they are also called communication satellites.

**Essential conditions for geostationary satellite.**

1. A geostationary satellite should be at a height nearly 36000 km above the equator of earth.
2. Its period of revolution around the earth should be the same as that of the earth about its axis i.e. exactly 24 hours.
3. It should revolve in an orbit concentric and coplanar with the equatorial plane, so the plane of orbit of the satellite is normal to the axis of rotation of the earth.
4. Its sense of rotation should be the same as that of the earth about its own axis i.e. from west to east. Its orbital velocity is nearly 3.1 km/s.

**Polar Satellite**

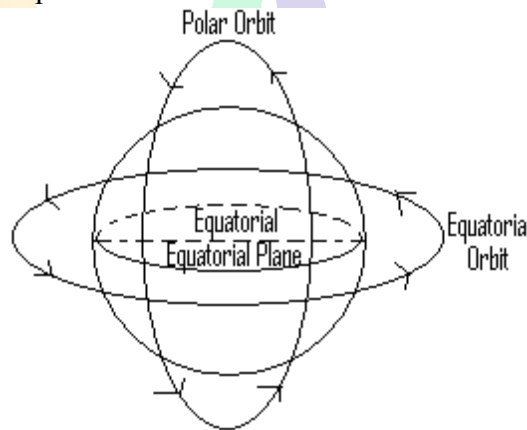
It is that satellite which revolves in polar orbit around earth. A polar orbit whose angle of inclination with equatorial plane of earth is  $90^\circ$  and a satellite in polar orbit will pass over both the north and south geographic poles once per orbit. Polar satellites are sun-synchronous satellites.

It is important to note that a single polar satellite can monitor 100% earth's surface. Every location on earth within the observation of polar satellite twice each day.

The geostationary environmental polar satellites are used for longer term forecasting.

The polar satellites are used for getting the cloud images, atmospheric data, ozone hole over Antarctica.

There are large number of polar satellites of different countries, which are active around earth. Seastar is one of the polar satellites

**Important Uses of Satellites**

1. In communicating radio, T.V. and telephone signals across the oceans,
2. In forecasting weather,
3. In studying the upper region of atmosphere,
4. To determine the exact shape and dimensions of earths,
5. In the study of cosmic rays and solar radiations.